Allocating Positional Goods: A Mechanism Design Approach*

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Abstract

Consumers of positional goods care about their relative positions in the consumption of the goods, so allocating an item to one buyer has externalities on others. Using a mechanism design approach, I characterize the externalities by a feasibility condition. I find the revenue-maximizing mechanism excludes some low types and fully separates the rest if and only if the buyer's type distribution is regular. The seller can guarantee at least half the maximal revenue by offering one level of positional goods, and the approximation can be arbitrarily close if the distribution is sufficiently concave. Moreover, if the distribution has an increasing (decreasing) failure rate, total pooling (full separation) without exclusion maximizes the consumer surplus, and the consumer surplus is decreasing (increasing) in the number of positional good levels. Applications include education, priority services, luxury goods, and organizational design.

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"*If everyone stands on tiptoe, no one sees better.*"

—Fred Hirsch (1976), *Social Limits to Growth*

1 Introduction

Positional goods are goods or services whose value to consumers depends on their relative positions in consumption. Hirsch (1976) first coined the concept of "positional good" that is "either scarce in some absolute or socially imposed sense or subject to congestion or crowding through more extensive use" (pp. 27). Examples of positional goods abound. Yet in most of them, competition among consumers (and even exclusion of some) seem to be beneficial to the seller but detrimental to consumer welfare.

Education is a classic example. According to Hirsch, "The value to me of my education depends not only on how much I have but also on how much the man ahead of me in the job line has." In East Asian countries, education arms races are particularly intense as numerous students compete for limited college and job slots.[1] While the competition increases the money and effort spent on education, it can be detrimental to the well-being of students and their families. Thus, the governments have implemented policies to orchestrate educational disarmament (The Economist, 2021). Similarly, to relieve student stress, many U.S. colleges switched to coarse grading, such as pass/fail, during the COVID-19 pandemic.

The crowding or congestion nature of positional goods is also epitomized by priority services in queuing (Gershkov and Winter, 2023), such as priority boarding. These services reduce consumers' waiting time by getting them ahead of the crowd but become less effective as more consumers purchase the service. In the extreme, when everyone buys the service, they are back to the original waiting time despite the extra money they spend. In addition, this zero-sum game can result from zero-sum thinking, a prevalent mindset that perceives one person's gain as another's loss (Bergeron et al., 2023).

Many consumer goods are positional because people derive happiness from comparisons (Frank, 1985a; Luttmer, 2005). For example, the value of luxury goods (e.g., cars and bags) is largely derived from the status they confer on the consumer relative to others (Carlsson et al., 2007). Luxury companies often have a proliferation of products (e.g., from factory store to high-end) and maintain their status value by limiting the supply and even destroying unsold goods.[2] While the status value benefits sellers, for

[1] Even in European countries where overeducation is less common as children are sorted into academic and vocational tracks (Di Stasio et al., 2016), education still seems to be positional (Durst, 2021).

[2] "Burberry burns bags, clothes and perfume worth millions" (BBC, 2018).

the individual (consumer) well-being, luxury consumption and "keeping up with the Joneses" are disfavored in certain cultures and religions (e.g., Buddhism).[3]

More broadly, status within an organization can also be viewed as a positional good, as employees invest effort to move up in the hierarchy (Moldovanu et al., 2007). Just as a revenue-maximizing seller, the organization designer can also induce effort through employees' positional concerns. In addition, some organizations are known to terminate employees who underperform (e.g., "rank-and-yank").[4] While the practice does incentivize performance, it remains controversial as it leads to employee welfare loss. On a larger scope, some cultures and philosophies also advocate the elimination or reduction of status classes in the society to enhance individual well-being.

In all these examples, consumption of positional goods has *externalities* on others, as moving up the position of one consumer would inevitably push down that of another. Unlike ordinary goods, it is impossible to allocate the highest status to all consumers at face value: "What each of us can achieve, all cannot" (Hirsch, 1976). Thus, positional goods induce a game of status, in which consumers (or students/employees) pay in money (or through effort) for a higher relative position or status.[5]

In this paper, I study the allocation of positional goods using a mechanism design approach and consider the seller's revenue (total effort), consumer surplus (agent welfare), and exclusion. I assume the buyers care about their relative positions or *status*, defined by the mass of consumers of strictly lower-level goods plus one-half of consumers at the same level. I characterize positional externalities by a feasibility condition that the interim status is a mean-preserving spread of the buyer's type distribution in the quantile space. Using results in Kleiner et al. (2021, henceforth KMS), I find the revenue-maximizing mechanism excludes some low types with negative marginal revenue and separates the remaining types if and only if the type distribution is regular. The result resembles the standard results in selling ordinary (nonpositional) goods (e.g. Mussa and Rosen, 1978) and is robust to the introduction of varying intrinsic quality of positional goods produced at heterogeneous costs.

I also find that a simple mechanism that offers one level of positional goods at a fixed price can guarantee at least half the maximal revenue. In the proof, I considered an auxiliary screening problem of selling a continuum of ordinary (nonpositional) goods

[3]Economists, dating back to John Stuart Mill, have also suggested the use of luxury taxes to correct for the externalities (see also Frank, 2005, 2008).

[4]"Rank and yank" or stack ranking is a management practice to rank every employee and then fire the bottom 10%. The practice was used by large companies including Microsoft, General Electric and Accenture Consulting, and is still in use in Amazon and IBM (Kantor and Streitfeld, 2015).

[5]Because of the dual interpretation of the model, I use consumers and agents (students and employees) interchangeably. Sellers and principals (organization designers) are also used interchangeably.

to a continuum of buyers. The screening problem highlights the distinction between selling positional and nonpositional goods: in the former, allocating the good to one buyer has externalities on others, so the seller cannot sell the highest status to multiple buyers at face value. Therefore, the seller's revenue is lower when selling positional goods. I also show this posted-price mechanism can perform arbitrarily closely to the revenue-maximizing mechanism if the distribution is sufficiently concave.

As for the consumer surplus, offering more positional levels, which lead to a coarser partition, increases the consumer surplus if the buyer's type distribution has an increasing failure rate (IFR), and decreases the consumer surplus if and only if the distribution has a decreasing failure rate. Thus, under the IFR property, total pooling maximizes the consumer surplus.[6] In contrast to allocating ordinary (i.e., nonpositional) goods, it is unclear *a priori* whether exclusion is detrimental to the consumer surplus, as it increases the status of the remaining consumers who would be potentially pooled with the excluded ones otherwise. This benefit of exclusion is not present in the screening problem, where goods can be allocated to every buyer at face value because there are no externalities. Nevertheless, I show that under the IFR property, the loss of the excluded consumers exceeds the gain of the remaining ones. On the other hand, when the distribution has a decreasing failure rate, the consumer surplus is maximized by full separation and no exclusion.[7]

The model has dual interpretations, which lead to two strands of applications. On the one hand, it is a monopolist problem of selling positional goods, such as priority services (Gershkov and Winter, 2023) and luxury goods. The monopolist problem is akin to selling ordinary goods (e.g., Mussa and Rosen, 1978) but different due to positional externalities; and it is also closely related to the monopolist problem of conspicuous goods (Rayo, 2013). Alternatively, it can be interpreted as a designer allocating status within an organization to motivate agent effort (e.g., Moldovanu et al., 2007; Immorlica et al., 2015) or maximize agent welfare. Under this interpretation, the optimal mechanism that maximizes effort (which is the same as revenue under linear effort costs) excludes low types and fully separates the participants if the type distribution is regular. This is the case in "rank-and-yank" organizations that rank every employee and terminate the underperforming ones. The interpretation also echoes the original example of education by Hirsch (1976). From the agent's welfare perspective, while the common wisdom is that educational disarmament by inducing coarser partitions (in particular, total pooling) is better, I show

[6]The necessary and sufficient condition is also provided in the paper.

[7]When there is full separation, exclusion always reduces the consumer surplus because it has no benefit on the remaining consumers.

that this is true if the type distribution satisfies the IFR property. On the contrary, if the distribution has a decreasing failure rate, full separation maximizes agents' surplus. Intuitively, when there are a few high types at the very top (e.g., Pareto distribution), it is better to separate them even from the agent's perspective. My results also explain how a different track of education (e.g., vocational versus academic education) mitigates positional externalities in education, as it essentially introduces a new set of positional goods that is incomparable with the other.

In Appendix A, I consider the case where exclusion is impossible as the lowest status is available for free. In contrast to being excluded, getting the lowest-level positional good can have a positive value if many types are pooled at this level—the lowest status cannot to offered to multiple agents at face value either. This case corresponds to contests for status in Moldovanu et al. (2007) (as investing no effort guarantees the lowest status) and monopoly provision of priority services in Gershkov and Winter (2023) (as the nonpriority service is free). I provide the necessary and sufficient conditions for some of their results and show some additional results. These results are qualitatively similar to the case with exclusion.

Literature Review. The idea that the consumer's utility depends on the comparison with others' consumption dates back to Veblen (1899) (see also Duesenberry, 1949; Leibenstein, 1950; Bagwell and Bernheim, 1996). Since Hirsch (1976) introduced the concept of positional good, the status interdependence in preference has been formally modeled and studied by Frank (1985b, 2005, 2008), Robson (1992) and Hopkins and Kornienko (2004).[8] This literature focuses on consumer choice in the presence of positional externalities and the welfare effects of changes in income distribution and government policies (e.g. luxury tax).

Empirical evidence of the importance of relative positions include Frank (1985a), Luttmer (2005), Alpizar et al. (2005), Carlsson et al. (2007). See a survey of the empirics and models of social status by Truyts (2010).[9] In particular, education is a typical example of positional good (Hirsch, 1976). Durst (2021) finds empirical evidence that education is indeed positional in Germany. Di Stasio et al. (2016) find that education is less positional in countries with more developed vocational education systems (e.g., Germany).

Moldovanu et al. (2007, henceforth MSS) study the optimal design of the number

[8]Another strand of literature uses the Duesenberry formulation that stems from Duesenberry (1949) and Pollak (1976) (see the survey by Truyts (2010)), which assumes the consumer's utility depends on the consumption of others in addition to their absolute consumption.

[9]See also Pagano (1999); Pagano and Vatiero (2019) for law and economics and Brighouse and Swift (2006) for sociology.

and size of status categories in an organization to maximize agents' effort when they care about their relative position (i.e., status). They find that full separation is optimal if the ability distribution satisfies the IFR property, while coarse partition may be optimal depending on the distribution—for example, the optimal partition involves only two categories if it is sufficiently concave. Despite the similarities in our results, this paper is different in three ways.[10] First, I allow for exclusion of agents and stochastic allocations and study agent welfare maximization. In reality, exclusion of agents is common in organizations (e.g., stack ranking), and employee welfare is an important consideration. Second, in the case without exclusion, I obtain stronger results that are either necessary and sufficient conditions or require fewer distributional assumptions (see Appendix A). Third, I use a mechanism design approach with a feasibility condition that applies the results by KMS, while they use the optimal contest design (with finite agents).

Relatedly, Rayo (2013) studies a model of monopolist provision of conspicuous goods and characterizes the optimal allocation. His results are qualitatively similar to MSS's, and he does not consider exclusion, consumer welfare maximization, or stochastic mechanisms either. Conspicuous goods in his definition also differ from positional goods (or status in MSS) in that consumers care about the average type in their assigned category instead of relative positions. Nevertheless, the feasibility condition can also be applied to conspicuous goods in his model (see Remark 3), which explains the similarities between his results and MSS's.[11]

Similar to MSS, Immorlica et al. (2015) study the optimal design of badges to incentivize users' contribution (i.e., effort) to the online community through the status value. Unlike MSS, they consider exclusion and allow the status value to be concave or convex in the relative position. They find the "leaderboard with a cutoff" mechanism is optimal if the distribution is regular. Although the result is consistent with mine, there are several differences. First, as in MSS, they do not consider user welfare or stochastic allocations. In online communities, user welfare is still an important consideration, and badges may be randomly assigned. Second, their status value function is strictly decreasing in the number of people *weakly* superior to them (without adjustment to those at the same level), which makes pooling of users unlikely to be optimal because it decreases the status of every user pooled together. Consequently, (i) total pooling will never be optimal even

[10] The definition of status in MSS, which equals the number of strictly inferior agents minus the number of strictly superior agents, is different but essentially equivalent to my specification (see Footnote 13).

[11] Board (2009) assumes more general benefits from being in a category (i.e., group quality) and finds a contradictory result that the revenue-maximizing principal segregates agents too finely compared to the efficient level (which is full separation in MSS, Rayo (2013), and my model). He also finds revenue maximization leads to too much exclusion.

if they consider users' welfare, in sharp contrast to my result; (ii) the optimality of full separation in effort maximization is also less surprising; and (iii) exclusion is equivalent to pooling with the lowest status. Third, they use Bernstein polynomial approximation of the status function instead of the feasibility characterization in terms of mean-preserving spreads.[12] Moreover, they focus on the approximation through threshold mechanisms and leaderboard mechanisms without cutoff.

A closely related paper, although in a different context, is by Gershkov and Winter (2023, henceforth GW), who study a model of priority service and analyze its implications on consumer welfare. Priority service (PS) is an example of positional goods arising from technological or capacity constraints. Using a mechanism design approach, this paper generalizes their model with a monopoly PS provider by allowing for multiple (and stochastic) priority levels and exclusion. By contrast, they focus on two PS levels (except for Section V, which does not consider exclusion) instead of the optimal mechanism and do not consider exclusion when there are multiple PS levels. In the case without exclusion (see Appendix A), I provide the necessary and sufficient conditions for some of their results and show some additional results.

Broadly, this paper is also related to externalities in mechanism design (Jehiel et al., 1996; Jehiel and Moldovanu, 2005).

2 Model

2.1 Setup

A seller (she) sells positional goods $q \in L \subseteq \mathbb{R}_{++}$ to a continuum of buyers (he) with unit mass. Positional good induces a game of status as buyers care about their relative position in consumption. Denote by G the distribution of buyers' equilibrium consumption of the good, so $G(q)$ represents the mass of buyers who consume (weakly) lower-level goods than q. When a buyer consumes a positional good q, his relative position or *status* is given by

$$S(q) = \Pr(\tilde{q} < q) + \Pr(\tilde{q} = q)/2 = \frac{G^{-}(q) + G(q)}{2} \in [0, 1],$$

where $G^{-}(q) \equiv \lim_{q \to q^{-}} G(q)$ is the mass of buyers who consume *strictly* lower-level goods than q. In other words, the status equals the mass of buyers who consume strictly lower-

[12]The feasibility characterization and subsequent results are robust to the convexity or concavity of valuations in the relative position (see Remark 3 and 5). They can also be extended to their specification where the pooling decreases total status (see Remark 4 and 7).

level goods plus one-half of the mass of buyers at the same level.[13] The specification of status can arise from psychological reasons (Robson, 1992; Hopkins and Kornienko, 2004) in that buyers derive utility from the mass of consumers of lower-level goods, but not as much from those of the same level. It can also be due to technological reasons (Gershkov and Winter, 2023) in that consumers of higher positions are served earlier and wait less time, while consumers of the same level are served in random order. Importantly, the total status is always a constant, corresponding to the "zero-sum" nature of positional goods that can also arise from zero-sum thinking.

Buyers are heterogeneous in their valuations (i.e., types) of the status, denoted by θ. Assume the buyer's type θ has a distribution $F(\theta)$ with support $[0, \bar{\theta}]$ and density $f(\theta) > 0$.[14] A buyer with type θ who pays $p \geq 0$ for a status $s \in [0, 1]$ receives a payoff of $u(p, s, \theta) = \theta(s + \alpha) - p$, where $\alpha \geq 0$ is a constant representing the lowest possible status, as in Hopkins and Kornienko (2004), relative to the reservation utility (from not buying) normalized to 0. Thus, α can be interpreted as the intrinsic quality of the positional good. For example, a luxury good has both intrinsic value and status value arising from comparison with others. I assume every positional good has the same intrinsic quality to focus on the role of its status value and abstract from the problem of quality differentiation (Mussa and Rosen, 1978; Maskin and Riley, 1984). In practice, positional goods differ in intrinsic quality, but the differences are insignificant compared to their status value (see Carlsson et al., 2007).[15] In Section 4, I consider an extension where the positional good q also has an intrinsic quality equal to q (so that $u(p, s, \theta, q) = \theta(s + q + \alpha) - p$) and is produced at a cost $c(q)$.

If exclusion is not allowed, the intrinsic quality α does not matter (and can be negative if the status game is assumed to be a necessary suffering). In this case, to restrict the price the seller can charge, it is typically assumed that the lowest-level positional good is free, which is the buyer's outside option but different from being excluded (see MSS and Section 2 in GW). With exclusion, buyers who cannot afford any positional good

[13]This specification ensures that the total status is constant $(1/2)$ for any distribution G, regardless of mass points due to pooling. An essentially equivalent specification à la MSS and Dubey and Geanakoplos (2010) is

$$S(q) = \Pr(\tilde{q} < q) - \Pr(\tilde{q} > q) = G(q) + G^{-}(q) - 1,$$

which results in a total status of 0.

Hopkins and Kornienko (2004) assume a more general specification, $S(q) = \gamma G(q) + (1 - \gamma)G^{-}(q)$ with $\gamma \in [0, 1)$, so that the total status is not necessarily constant (see also Immorlica et al. (2015) for $\gamma = 0$), which I will discuss later.

[14]The support can be extended to $\bar{\theta} = \infty$ and $\underline{\theta} > 0$. The latter case leads to complications in consumer surplus maximization when exclusion is impossible (see Appendix A.2, Remark 10).

[15]Theoretically, the seller can create heterogeneous status by offering products with infinitesimal differences in intrinsic quality at different prices.

receive the reservation utility zero and are considered to be strictly below every consumer of the good in the status calculation. In other words, if the buyer consumes the good $q \in L \subseteq \mathbb{R}_+$, his status $S(q)$ always includes the mass of excluded buyers.[16] Formally, denote not buying by $0 \notin L \subseteq \mathbb{R}_{++}$, and then extend the domain of $G(q)$ to $L \cup \{0\}$ so that $G(0)$ is the mass of the excluded buyers. Thus, the status is still $S(q) = (G(q) + G^-(q))/2$ for all $q \in L$ as defined above; for nonparticipants (who are assigned to $q = 0$), define $S(0) = -\alpha$ so that they receive zero utility. Because I assume $\alpha \geq 0$ relative to the reservation utility, the positional good has a positive intrinsic value.

Alternatively, à la MSS, one can interpret buyers as agents who care about their relative positions within the organization, and the monopoly seller as a principal maximizing effort by designing the number and size of status levels. Under this interpretation, the price p represents the agent's effort, the type θ represents the agent's ability that determines his marginal cost of effort, and α represents the lowest status within the organization. The agent has linear cost of effort p/θ, and his payoff is $(s + \alpha) - p/\theta$, which is equivalent to the original payoff $u(p, s, \theta)$. As in the case of exclusion, when the agent is expelled from the organization, he receives zero utility ($s = -\alpha$).

2.2 A Mechanism Design Approach

Consider a direct mechanism $\{p(\theta), q(\theta, \omega)\}$, consisting of a payment function $p\colon \Theta \to \mathbb{R}_+$ and a (potentially stochastic) positional good allocation $q\colon \Theta \times \Omega \to L$, where the random variable $\omega \in \Omega = [0, 1]$ captures the randomness in the mechanism.[17] Equivalently, it is without loss to consider a direct mechanism $\{p(\theta), s(\theta)\}$, where $s\colon \Theta \to [0, 1]$ is the interim status induced by the allocation q, given by $s(\theta) = \mathbf{E}_\omega[S(q(\theta, \omega)) \mid \theta]$. In particular, under a deterministic allocation $q\colon \Theta \to L$, the interim status is $s(\theta) = S(q(\theta))$.

Denote $U(\theta) = \theta(s(\theta) + \alpha) - p(\theta)$. For excluded buyers, $s(\theta) = -\alpha$ and $p(\theta) = 0$ so that they receive zero utility.

Lemma 1. *A direct mechanism $\{p(\theta), s(\theta)\}$ is incentive-compatible if and only if*

- $U(\theta) = U(0) + \int_0^\theta (s(x) + \alpha)\,\mathrm{d}x$ *for all* $\theta \in [0, \bar{\theta}]$*, and*
- $s(\theta)$ *is increasing.*

[16] This is more obvious by writing the status as $S(q) = 1 - \Pr(\tilde{q} > q) + \Pr(\tilde{q} = q)/2$, i.e., the total mass (1) minus the mass of buyers who consume strictly higher-level goods plus one-half of the mass of consumers of the same-level good.

[17] The allocation can be implemented by a pricing scheme as a function of the (distribution of) positional good levels.

Lemma 2. *There exists a cutoff type* $\hat{\theta} \in [0, \bar{\theta}]$ *such that the buyer participates (i.e.,* $U(\theta) \geq 0$*) if and only if* $\theta \geq \hat{\theta}$*.*

An increasing interim status function $s(\theta)$, albeit incentive-compatible, is not necessarily feasible because it may be unable to be induced by an allocation q. Formally, say an interim status $s(\theta)$ is *feasible* if there exists an allocation q that induces it, i.e., $s(\theta) = \mathbf{E}_\omega[S(q(\theta, \omega)) \mid \theta]$. In particular, for a deterministic allocation $q(\theta)$, incentive compatibility implies $q(\theta)$ is increasing because $s(\theta) = S(q(\theta))$ is increasing. Thus, $s(\theta) = S(q(\theta)) = \mathbf{E}[F(\tilde{\theta}) \mid q(\tilde{\theta}) = q(\theta)]$. To see this, if $q(\theta)$ is strictly increasing at θ, then $s(\theta) = F(\theta)$. Otherwise, if buyers on an interval $[\theta_1, \theta_2]$ consume the same good q_1 (i.e., $q(\theta) = q_1$ if and only if $\theta \in [\theta_1, \theta_2]$), then

$$s(\theta) = \frac{G(q_1) + G^{-}(q_1)}{2} = \frac{F(\theta_1) + F(\theta_2)}{2} = \mathbf{E}[F(\tilde{\theta}) \mid \tilde{\theta} \in [\theta_1, \theta_2]] = \mathbf{E}[F(\tilde{\theta}) \mid q(\tilde{\theta}) = q(\theta)]$$

for $\theta \in [\theta_1, \theta_2]$. Thus, one can also write $S(q) = \mathbf{E}[F(\theta) \mid q]$.

Now I characterize the necessary and sufficient condition for an increasing (and thus incentive-compatible) status $s(\theta)$ to be feasible without restriction to deterministic allocations.

Theorem 1 (Feasibility)**.** *An increasing* $s(\theta)$ *is feasible if and only if it is a mean-preserving spread of* $F(\theta)$ *in the quantile space on* $[\hat{\theta}, \bar{\theta}]$ *for some* $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$*, denoted by* $s \in \mathrm{MPS}(F \cdot \mathbf{1}_{[\hat{\theta}, \bar{\theta}]})$*, that is,*

$$\int_x^{\bar{\theta}} s(\theta)\, \mathrm{d}F(\theta) \leq \int_x^{\bar{\theta}} F(\theta)\, \mathrm{d}F(\theta) \text{ for all } x \in [\hat{\theta}, \bar{\theta}], \tag{1}$$

$$\int_{\hat{\theta}}^{\bar{\theta}} s(\theta)\, \mathrm{d}F(\theta) = \int_{\hat{\theta}}^{\bar{\theta}} F(\theta)\, \mathrm{d}F(\theta) = \frac{1 - F(\hat{\theta})^2}{2}. \tag{2}$$

Proof sketch. À la Kleiner et al. (2021, Theorem 3). As noted above, full separation (assortative matching) leads to $s(\theta) = F(\theta)$, while pooling on $[\theta_1, \theta_2]$ leads to

$$s(\theta) = \frac{1}{F(\theta_2) - F(\theta_1)} \int_{\theta_1}^{\theta_2} F(\theta)\, \mathrm{d}F(\theta) = \frac{F(\theta_1) + F(\theta_2)}{2}.$$

In particular, given the exclusion level $\hat{\theta}$, pooling of all participants leads to $s(\theta) = \frac{1+F(\hat{\theta})}{2}$.

($\Longrightarrow$) follows from $\mathbf{E}[\frac{1+F(\hat{\theta})}{2} \mid \theta \leq \tau] \leq \mathbf{E}[s(\theta) \mid \theta \geq \tau] \leq \mathbf{E}[F(\theta) \mid \theta \geq \tau]$ for all $\tau \in [\hat{\theta}, \bar{\theta}]$ (because switching to assortative matching takes the status of low type to high types). ($\Longleftarrow$) is by applying Choquet's Theorem to the extreme points of $\mathrm{MPS}(F \cdot \mathbf{1}_{[\hat{\theta}, \bar{\theta}]})$ (i.e., pooling or fully separating). □

Remark 1. For deterministic allocations, $s(\theta)$ must be an extreme point of $\mathrm{MPS}(F)$, i.e., either the majorization constraint or the monotonicity constraint binds (see Theorem 1 in KMS). For stochastic allocations, $s(\theta)$ can take other forms.[18]

Remark 2. When exclusion is impossible, the equality must hold at $x = 0$, that is, $\mathbf{E}[s] = \mathbf{E}[F] = 1/2$. Under the (especially equivalent) specification $S(q) = G^{-}(q) - (1 - G(q))$ as in MSS, the feasibility condition without exclusion becomes: an increasing $s(\theta)$ is feasible if and only if $s \in \mathrm{MPS}(2F - 1)$ in the quantile space, with $\mathbf{E}[s] = \mathbf{E}[2F - 1] = 0$.

Remark 3. In the model of conspicuous goods by Rayo (2013), where the status value $S(q) = \mathbf{E}[\theta \mid q]$, the feasibility condition (without exclusion) is $s \in \mathrm{MPS}(\theta)$ in the quantile space. More generally, for a strictly increasing function ϕ, if the status value is $S(q) = \mathbf{E}[\phi(F(\theta)) \mid q]$, the feasibility condition is $s \in \mathrm{MPS}(\phi \circ F)$ in the quantile space.[19]

Intuitively, under full separation, $s(\theta) = F(\theta)$. Any (partial) pooling of different types into the same positional good makes $s(\theta)$ a mean-preserving spread of $F(\theta)$ in the quantile space because the value of the positional good becomes lower than its face value (i.e., its value under full separation).

Example. Suppose there are two positional good levels, $\{p_L, s_L\}$ and $\{p_H, s_H\}$, where $p_L = 0$ (so there is no exclusion). Denote by θ^* the cutoff type whose is indifferent between the two. Then, the status is $s(\theta) = s_L = F(\theta^*)/2$ for $\theta < \theta^*$ and $s(\theta) = s_H = (1 + F(\theta^*))/2$ for $\theta \geq \theta^*$. It is easy to check that $\mathbf{E}[s] = 1/2$ and that $\int_\theta^{\bar{\theta}} s(\theta') \, \mathrm{d}F(\theta') \leq \int_\theta^{\bar{\theta}} F(\theta') \, \mathrm{d}F(\theta')$ for all $\theta \in [0, \bar{\theta}]$ (with equality at 0, θ^*, and $\bar{\theta}$).

3 Optimal Mechanisms

Two main objectives, revenue maximization and consumer surplus maximization, are considered in this section. Two extreme mechanisms are particularly of interest: full separation (i.e., $s(\theta) = F(\theta)$) and total pooling among participants (i.e., $s(\theta) = (1 + F(\hat{\theta}))/2$). The former can be induced by offering a continuum of positional goods that lead to a proliferation of status (i.e., the finest partition), and the latter can be induced by offering only one level of positional good (i.e., the coarsest partition).

[18] One can also write $s(\theta) = \mathbf{E}_{q \sim q(\theta, \Omega)}[\mathbf{E}[F(\tilde{\theta}) \mid q]]$, where $q(\theta, \Omega)$ denotes the distribution of the $q(\theta, \omega)$ generated by the randomness ω.

[19] This can be useful when the status value is a strictly increasing (convex or concave) function of the current specification (see, e.g., Immorlica et al., 2015).

3.1 Revenue Maximization

The revenue (or effort) maximization problem is given by

$$\max_{s(\theta),p(\theta),\hat{\theta}} \int_0^{\bar{\theta}} p(\theta)\,\mathrm{d}F(\theta) \tag{3}$$

subject to the following constraints:

$$
\begin{aligned}
& p(\theta) \geq 0 && && (4)\\
& U(\theta) \equiv \theta(s(\theta)+\alpha) - p(\theta) \geq 0 && \text{(IR)} && (5)\\
& U(\theta) = U(0) + \int_0^{\theta} (s(x)+\alpha)\,\mathrm{d}x && \text{(IC)} && (6)\\
& s(\theta) \text{ is increasing} && && (7)\\
& s \in \mathrm{MPS}(F \cdot \mathbf{1}_{[\hat{\theta},\bar{\theta}]}) && \text{(MPS)} && (8)
\end{aligned}
$$

and $s(\theta) = -\alpha$ for all $\theta < \hat{\theta}$.

Denote $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$. The revenue is given by

$$R = \int_0^{\bar{\theta}} J(\theta)s(\theta)\,\mathrm{d}F(\theta) - U(0) = \int_{\hat{\theta}}^{\bar{\theta}} J(\theta)(s(\theta)+\alpha)\,\mathrm{d}F(\theta) - U(0). \tag{9}$$

By $U(0) = -p(0) \geq 0$ (IR), it is optimal to set $U(0) = 0$ and $p(0) = 0$. The constraint $p(\theta) \geq 0$ is satisfied because $p(\hat{\theta}) = \hat{\theta}(s(\hat{\theta})+\alpha) \geq 0$ and $p'(\theta) = \theta s'(\theta) \geq 0$.

For the convenience of notations, the MPS constraint can be relaxed to $s \in \mathrm{MPS}_w(F)$ because the maximum is always attained at its extreme point in $\mathrm{MPS}(F \cdot \mathbf{1}_{[\hat{\theta},\bar{\theta}]})$ for some $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$, according to Corollary 2 in KMS. Thus, the revenue maximization problem is equivalent to

$$\max_{s\in\mathrm{MPS}_w(F)} \int_0^{\bar{\theta}} J(\theta)s(\theta)\,\mathrm{d}F(\theta). \tag{10}$$

The following proposition follows immediately from Theorem 4 (Fan-Lorentz) in KMS.

Proposition 2. *Assume $J(\theta)$ is increasing. The revenue-maximizing mechanism excludes types below $\hat{\theta}^* = J^{-1}(0) \in (0, \bar{\theta})$ and fully separates types above $\hat{\theta}^*$.*[20]

Remark 4. Under the specification $\tilde{S}(q) = \gamma G(q) + (1-\gamma)G^{-}(q)$ with $\gamma \in [0, 1/2]$, pooling is less likely to be optimal because it reduces total status value—if $\gamma < 1/2$, the interim status

[20] When there are multiple $\hat{\theta}$ such that $J(\hat{\theta}) = 0$, $\hat{\theta}^*$ can be any of them.

lies below $\mathrm{MPS}(F \cdot 1_{[\hat{\theta},\bar{\theta}]})$ and strictly below whenever there is pooling.[21] Thus, Proposition 1 still holds for $\gamma \in [0, 1/2]$. The linear case in Immorlica et al. (2015) corresponds to $\gamma = 0$.

Remark 5. Proposition 1 also holds when the status value is a strictly increasing function (regardless of convexity) of the current specification (Immorlica et al., 2015) because the only change is that $s \in \mathrm{MPS}_w(\phi \circ F)$ (see Remark 3), which makes no differences.

This result resembles the revenue-maximizing mechanism in selling ordinary (non-positional) goods (e.g. Mussa and Rosen, 1978), where agents with negative marginal revenue (i.e., $J(\theta) < 0$) will also be excluded. If $J(\theta)$ is not increasing in some regions, the ironing technique can be applied, and types in this region are pooled.

The mechanism can be implemented by an all-pay auction with a reserve price, where the higher the buyer pays (above the reserve price), the higher-level positional good he receives. In applications such as education or social status, it can be interpreted as an all-pay contest in which those whose effort does not meet the reserve price are punished with exclusion (i.e., zero status value).[22]

The result has two practical implications. First, in the absence of the status value (i.e., with intrinsic value $\alpha \geq 0$ only), the seller's revenue is $\alpha(1 - F(\hat{\theta}^*))\hat{\theta}^*$. By attaching the status value $s^*(\theta) = F(\theta)$ to the types, the seller can extract $\int_{\hat{\theta}^*}^{\bar{\theta}} J(\theta)s^*(\theta)\,\mathrm{d}F(\theta) > 0$ more revenue. For uniform distribution on $[0, 1]$, this is \$0.21 more than the maximal revenue \$$\alpha/4$ with pure intrinsic value; the difference can be substaintial if the intrinsic value α is low. This explains why luxury companies spend a huge amount on marketing.

Second, by excluding low types $\theta < \hat{\theta}^*$, the seller can extract $-\int_0^{\hat{\theta}^*} J(\theta)F(\theta)\,\mathrm{d}F(\theta) > 0$ more revenue than the case without exclusion. For uniform distribution on $[0, 1]$, it is \$0.04 more than the maximal revenue \$0.17 without exclusion. Thus, exclusion is important for the seller (organization designer) to extract more revenue (effort).

Proposition 3. *The seller can obtain at least half the maximal revenue by selling one level of positional good.*

Proof. The cutoff type indifferent between two levels is

$$\theta^*(p)\frac{1 + F(\theta^*(p))}{2} - p = -\alpha \implies p = \alpha + \theta^*(p)\frac{1 + F(\theta^*(p))}{2}.$$

[21] Formally, fix a positional good allocation q, the induced interim status $\tilde{s}(\theta) = \mathbf{E}_\omega[\tilde{S}(q(\theta,\omega)) \mid \theta]$ and $s(\theta) = \mathbf{E}_\omega[S(q(\theta,\omega)) \mid \theta]$ (in the original specification) satisfy $\tilde{s}(\theta) \leq s(\theta)$ for all $\theta \in [0, \bar{\theta}]$, with equality if and only if $\tilde{s}(\theta') = F(\theta')$ for all $\theta' \in [\hat{\theta}, \theta]$.

[22] See Section A for the case where such punishment is not possible.

Thus,

$$R_2 = \max_\theta \left(\alpha + \theta \frac{1 + F(\theta)}{2}\right)(1 - F(\theta)) \geq \frac{1}{2} \max_\theta \theta(1 - F(\theta))$$

because $\alpha \geq 0$ and $\frac{1+F(\theta)}{2}\theta \geq \frac{1}{2}\theta$.

Consider the auxiliary screening problem of selling an indivisible item of value to one buyer, in which a standard result implies a posted-price mechanism is optimal (see Börgers, 2015, Proposition 2.5). Formally, denote by q the probability the item is allocated to θ, and let $\mathcal{M} = \{q\colon [0, \bar{\theta}] \to [0, 1] \mid q \text{ increasing}\}$. Then, $\max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta)\, \mathrm{d}F(\theta) = \max_p p(1 - F(p))$, and any maximizer q^* must be an extreme point of $\mathcal{M}$ (i.e., a posted-price mechanism).

Because $\mathrm{MPS}(F) \subseteq \mathcal{M}$, we have

$$\max R = \max_{s \in \mathrm{MPS}_w(F)} \int_0^{\bar{\theta}} J(\theta)s(\theta)\, \mathrm{d}F(\theta) < \max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta)\, \mathrm{d}F(\theta) = \max_p p(1 - F(p)) \leq 2R_2.$$

The inequality is strict because an extreme point of $\mathcal{M}$ is not in $\mathrm{MPS}_w(F)$ (except when everyone excluded, which cannot be optimal). □

Remark 6. The result on the lower bound does not require the IFR property or the regularity condition.

Example. When $\alpha = 0$ and $F(\theta)$ is the uniform distribution on $[0, 1]$, the maximal revenue is $\max R \approx 0.21$, while the revenue from selling one level of positional good is $R_2 \approx 0.19$.

The auxiliary screening problem considered in the proof, which is equivalent to selling a continuum of indivisible ordinary (i.e., nonpositional) goods to a continuum of buyers, highlights the distinction between selling ordinary goods and positional goods. In the screening problem, selling an item to one consumer has no externalities on others. By contrast, negative externalities are present when selling positional goods—for example, the seller cannot offer the highest status to multiple buyers at face value. Thus, the seller of positional goods is subject to an additional constraint, resulting in (strictly) less revenue.

Proposition 4. *If $J(\theta)$ is single-dipped, there exists some $\hat{\theta} \in [0, \bar{\theta})$ and $\theta_p \in (\underline{\theta}, \hat{\theta})$ such that the revenue-maximizing mechanism excludes $\theta \in [0, \hat{\theta})$, pools $\theta \in [\hat{\theta}, \theta_p)$, and separates $\theta \in [\theta_p, \bar{\theta}]$.*

If $F(\theta)$ is sufficiently concave, then $J(\theta)$ is single-dipped, and the separating region $[\theta_p, \bar{\theta}]$ can be arbitrarily small.

Proof sketch. The first part follows from KMS. The second follows from $J'(\theta) = 2 + (1 - F(\theta))f'(\theta)/f(\theta)$. □

The proposition implies that offering one positional good level can perform arbitrarily closely to the revenue-maximizing mechanism if $F(\theta)$ is sufficiently concave.

3.2 Consumer Surplus Maximization

The consumer surplus (or agent welfare) maximization problem is given by

$$\max_{s(\theta),p(\theta),\hat{\theta}} \int_0^{\bar{\theta}} U(\theta)\,\mathrm{d}F(\theta) \tag{11}$$

subject to constraints (4)–(8).

For any given cutoff $\hat{\theta} \in [0, \bar{\theta}]$, the welfare maximization problem is equivalent to

$$\max_{s\in \mathrm{MPS}_w(F)} \int_0^{\bar{\theta}} \left(\frac{1-F(\theta)}{f(\theta)}\right)(s(\theta)+\alpha)\,\mathrm{d}F(\theta) + U(0) \tag{12}$$

Because $p(0) \geq 0$, $U(0) = -p(0) \geq 0$ (IR) implies $U(0) = p(0) = 0$.[23]

Proposition 5. *(i) Assume F satisfies the IFR property. The consumer surplus-maximizing mechanism has no exclusion and pools every type at the same status.*

(ii) Assume $\frac{1-F(\theta)}{f(\theta)}$ is increasing. The consumer surplus-maximizing mechanism has no exclusion and induces full separation.

Proof. (i) Under IFR, $\frac{1-F(\theta)}{f(\theta)}$ is decreasing, so by Corollary 2 in KMS, the optimal $s^*(\theta)$ pools all types $\theta \in [\hat{\theta}, \bar{\theta}]$, i.e.,

$$s^*(\theta) = (1 + F(\hat{\theta}))/2 \tag{13}$$

Now consumer surplus can be written as a function of the cutoff $\hat{\theta}$:

$$W(\hat{\theta}) = \int_{\hat{\theta}}^{\bar{\theta}} \frac{1-F(\theta)}{f(\theta)}[\alpha + 1/2 + F(\hat{\theta})/2]\,\mathrm{d}F(\theta), \tag{14}$$

with derivatives

$$W'(\hat{\theta}) = -(1-F(\hat{\theta}))(\alpha+1/2) - (1-F(\hat{\theta}))F(\hat{\theta})/2 + \frac{f(\hat{\theta})}{2}\int_{\hat{\theta}}^{\bar{\theta}} (1-F(\theta))\,\mathrm{d}\theta \tag{15}$$

[23] This would not be true without the assumption $p(\theta) \geq 0$. If negative transfers are allowed and subject to budget balance, full separation (assortative matching) will maximize consumer surplus because it is efficient, and transfers can redistribute the efficiency gains among consumers (see, e.g., Gershkov and Schweinzer, 2010). I thank Alex Gershkov for pointing this out.

It is straightforward that $W'(\bar{\theta}) = 0$ and $W''(\bar{\theta}) = (\alpha + 1)f(\bar{\theta}) \geq 0$. Denote

$$z(\hat{\theta}) \equiv -\frac{W'(\hat{\theta})}{1 - F(\hat{\theta})} = \alpha + 1/2 + F(\hat{\theta})/2 - \frac{f(\hat{\theta})}{1 - F(\hat{\theta})} \int_{\hat{\theta}}^{\bar{\theta}} \frac{1 - F(\theta)}{2} \, \mathrm{d}\theta, \tag{16}$$

Note that $z(\bar{\theta}) - F(\bar{\theta}) = \alpha \geq 0$, and under IFR,

$$z'(\hat{\theta}) - f(\hat{\theta}) = -\left[\frac{f(\theta)}{1 - F(\theta)}\right]' \int_{\hat{\theta}}^{\bar{\theta}} \frac{1 - F(\theta)}{2} \, \mathrm{d}\theta \leq 0.$$

Thus, $z(\theta) - F(\theta) \geq 0$, so $W'(\hat{\theta}) \leq 0$. Hence, the optimal cutoff is $\hat{\theta}^* = 0$.

(ii) If $\frac{1-F(\theta)}{f(\theta)}$ is increasing, $s^*(\theta) = F(\theta)$ for all $\theta \geq \hat{\theta}$, so

$$W(\hat{\theta}) = \int_{\hat{\theta}}^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)}\right) (F(\theta) + \alpha) \, \mathrm{d}F(\theta).$$

Therefore, no exclusion is optimal because $W'(\hat{\theta}) = -(1 - F(\hat{\theta}))(F(\hat{\theta}) + \alpha) \leq 0$. □

Remark 7. Under the specification $\tilde{S}(q) = \gamma G(q) + (1 - \gamma)G^{-}(q)$. The first (second) part of the proposition still holds if $\gamma \in [1/2, 1]$ ($\gamma \in [0, 1/2]$). In particular, if $\gamma = 0$ (Immorlica et al., 2015), full separation is always optimal.

When the mechanism pools all agents at the same status, they pay nothing as $p(\theta) = 0$ for all $\theta \in [0, \bar{\theta}]$. In the education application, this implies complete educational disarmament. Under the IFR property, the consumer surplus decreases with the number of status levels (see Proposition A.4).

If the failure rate $\frac{f(\theta)}{1-F(\theta)}$ is decreasing, because it is optimal to fully separate participants, it is obvious that no exclusion is optimal. However, under IFR, because it is optimal to pool participants, it is unclear *a priori* whether exclusion is detrimental to the consumer, as it increases the status of the participants (although at higher prices) who would be pooled with the excluded consumers otherwise. The proposition shows that no exclusion is still optimal in this case because the loss of the excluded consumers exceeds the gain of the remaining ones.

For distributions with nonmonotone failure rates, the ironing technique can be applied again to derive the optimal mechanisms, which can have pooling and separating regions.

3.3 Social Planner

Assume that the social planner maximizes a weighted sum of the seller's revenue and consumer surplus, so social welfare is $S = \int_0^{\bar{\theta}} \lambda p(\theta) + U(\theta)\, \mathrm{d}F(\theta)$, where $\lambda \geq 0$. In the monopolist provision problem where p is transferred to the seller, this simply means the social planner maximizes the weighted sum of the seller's revenue (at weight $\lambda/(1+\lambda)$) and consumer surplus. By contrast, in the education or the organization example, where p is essentially the agent's effort (so utilities are nontransferrable), λ captures the extent to which effort is productive relative to signaling. When $\lambda = 1$, agent effort is productive; as $\lambda \leq 1$ decreases to zero, it becomes more of signaling. When $\lambda > 1$, the social benefit of effort is higher than the private benefit.

By previous results,

$$S = \int_0^{\bar{\theta}} \left(\lambda\theta - (\lambda - 1)\frac{1 - F(\theta)}{f(\theta)} \right) s(\theta)\, \mathrm{d}F(\theta) + \alpha\, \mathbf{E}[\theta]. \tag{17}$$

Denote $J_\lambda(\theta) = \lambda\theta - (\lambda - 1)\frac{1-F(\theta)}{f(\theta)}$. Using the same arguments, we have the following results.

Proposition 6. *If $J_\lambda(\theta)$ is increasing, the social welfare-maximizing mechanism excludes types below $\hat{\theta}^* = J_\lambda^{-1}(0) \in [0, \bar{\theta})$ and fully separates types above $\hat{\theta}^*$.*

If $J_\lambda(\theta)$ is decreasing, the social welfare-maximizing mechanism pools every type.

Corollary 6.1. *If $\lambda = 1$, the social welfare-maximizing mechanism induces full separation and has no exclusion.*

Intuitively, when $\lambda = 1$, transfers are welfare-neutral. Hence, a finer partition strictly increases social welfare because it increases efficiency by matching the higher types (i.e., valuations for status) with higher status.

4 Intrinsic Quality

In this section, I consider the case where the positional good also has an intrinsic quality q produced at a cost $c(q)$. Assume $c(q)$ is strictly increasing and strictly convex. Denote by $Q(\theta) = \mathbf{E}_\omega[q(\theta, \omega)]$ the interim allocation. Thus, the buyer's utility is $U(\theta) = \theta(s(\theta) + Q(\theta) + \alpha) - p(\theta)$.

Lemma 3. *A direct mechanism $\{p(\theta), s(\theta), Q(\theta)\}$ is incentive-compatible if and only if*

- $U(\theta) = U(0) + \int_0^\theta (s(x) + Q(x) + \alpha)\, \mathrm{d}x$ *for all* $\theta \in [0, \bar{\theta}]$*, and*

- $s(\theta) + Q(\theta)$ *is increasing.*

Using the same arguments as before, the revenue maximization problem is

$$\max_{s \in \text{MPS}(F), Q} \int_0^{\bar{\theta}} [J(\theta)(s(\theta) + Q(\theta)) - c(Q(\theta))] \, \mathrm{d}F(\theta) \tag{18}$$

subject to $s(\theta) + Q(\theta)$ is increasing.

Proposition 7. *Assume $J(\theta)$ is increasing. The revenue-maximizing mechanism excludes types below $\hat{\theta}^* = J^{-1}(0)$ and offers $Q^*(\theta) = c'^{-1}(J(\theta))$ to $\theta \geq \hat{\theta}^*$.*

Proof. First show the monotonicity condition is equivalent to $s(\theta)$ or $Q(\theta)$ being increasing.

Lemma 4. *$s(\theta)$ is (strictly) increasing if and only if $Q(\theta)$ is (strictly) increasing. Thus, $s(\theta)$ (or $Q(\theta)$) is increasing if and only if $s(\theta) + Q(\theta)$ is increasing.*

Proof. Fix any $\omega \in \Omega$, by the definition of $S(\cdot)$, $S(q(\theta, \omega))$ is strictly increasing (constant) if and only if $q(\theta, \omega)$ is strictly increasing (constant). The same holds for $s(\theta) = \mathbf{E}[S(q(\theta, \omega)) \mid \theta]$ and $Q(\theta) = \mathbf{E}[q(\theta, \omega) \mid \theta]$. □

Because the exclusion level $\hat{\theta}^* = J^{-1}(0)$ in the standard case of pure intrinsic value coincides with that in the pure status value case in the previous section, the proposition follows directly from the lemma. □

The proposition implies that the optimal mechanism in revenue maximization are robust to the introduction of intrinsic quality. Moreover, by attaching the status value, the seller can extract $\int_{\hat{\theta}^*}^{\bar{\theta}} J(\theta) s^*(\theta) \, \mathrm{d}F(\theta) > 0$ more revenue. For uniform distribution on $[0, 1]$ and quadratic costs $c(q) = q^2/2$, the revenue is \$0.29 compared to \$0.08 in the pure status value case. Again, this explains the amount of money luxury companies spent on marketing (to create status value) instead of research and delvelopment (to reduce the cost).

5 Conclusion

In this paper, I study the allocation of positional goods that have externalities on others using a mechanism approach. The model has dual implications in the monopolist provision of positional goods and the design of status levels within organizations. In terms of the monopolist problem, the seller can extract more revenue from the consumers by offering many levels of positional goods through their status values, which is often detrimental to consumer surplus. In terms of the organizational design problem, as the common wisdom implies, the management practice of stack ranking (i.e., rank every employee and fire the bottom 10%) and rat races in education maximize agents' effort if their ability distribution is regular. Exclusion of low types can increase revenue or effort. On the contrary, from the agent's perspective, a coarser partition is beneficial if the ability distribution satisfies IFR. This explains the call for abolishing stack ranking and educational disarmament. Nevertheless, when there are a few high types at the very top (e.g., Pareto distribution), rat races can be better for agent welfare. Thus, the objectives to maximize agent effort and welfare can be aligned, which challenges the common wisdom that rat races are always detrimental to agents. In either case, exclusion always harms agent welfare, even though it can increase the status of the remaining agents.

It remains to be studied how competition among sellers affects the results. This can be interpreted as offering a different track of education that leads to different status levels (e.g., vocational versus academic education) or having another product that has a separate queue—in either case, consumers of one set of positional goods derive utility independently of those of the other, either because it is psychologically incomparable or physically separate. In the symmetric case, if the other seller offers the same menu as the original seller, an immediate result is that each seller will attract half of the buyers, and that competition can increase consumer surplus. However, when sellers compete in mechanisms strategically, the problem remains an open question.

A Optimal Mechanisms without Exclusion

In this section, I assume exclusion is impossible and assume the lowest position is free. This is the assumption made by Moldovanu et al. (2007), where agents cannot be excluded and will at least receive the lowest status, and the benchmark model of Gershkov and Winter (2023), where buyers can use the regular (non-priority) service for free. Using the mechanism design approach, I provide the necessary and sufficient conditions for some of their results and show some additional results. These results are qualitatively similar to the case with exclusion.

A.1 Revenue Maximization

The revenue maximization problem is given by

$$\max_{s(\theta),p(\theta)} \int_0^{\bar{\theta}} p(\theta)\,\mathrm{d}F(\theta) \tag{A.1}$$

subject to

$$p(\theta) \geq 0 \tag{A.2}$$

$$U(\theta) \equiv \theta(s(\theta)+\alpha) - p(\theta) \geq \theta(\underline{s}+\alpha) \qquad \text{(IR)} \tag{A.3}$$

$$\theta s(\theta) - p(\theta) = U(0) + \int_0^{\theta} s(x)\,\mathrm{d}x \qquad \text{(IC)} \tag{A.4}$$

$$s(\theta) \text{ is increasing} \tag{A.5}$$

$$s \in \mathrm{MPS}(F) \qquad \text{(MPS)} \tag{A.6}$$

where $\underline{s} = \min\{s(\theta)\}$ denotes the lowest status endogenously determined by the status allocation. Because $p'(\theta) = \theta s'(\theta) \geq 0$, the constraint $p(\theta) \geq 0$ can be reduced to $p(0) \geq 0$. The (IR) constraint can also be reduced to $U(0) \geq 0$ because $U'(\theta) - \underline{s} = s(\theta) - \underline{s} \geq 0$.

The revenue is given by

$$R = \int_0^{\bar{\theta}} p(\theta)\,\mathrm{d}F(\theta) = \int_0^{\bar{\theta}} \left(\theta - \frac{1-F(\theta)}{f(\theta)}\right) s(\theta)\,\mathrm{d}F(\theta) - U(0). \tag{A.7}$$

Because $U(0) = -p(0) \geq 0$ (IR), it is optimal to set $U(0) = p(0) = 0$. Denote $J(\theta) =$

$\theta - \frac{1-F(\theta)}{f(\theta)}$. The revenue maximization problem is equivalent to

$$\max_{s \in \mathrm{MPS}(F)} \int_0^{\bar{\theta}} J(\theta) s(\theta) \, \mathrm{d}F(\theta). \tag{A.8}$$

Proposition 2 and Corollary 2 in KMS imply the following results.

Proposition A.1. *A finer partition (strictly) increases the seller's revenue if and only if $J(\theta)$ is (strictly) increasing. Thus, the revenue-maximizing mechanism induces full separation if and only if $J(\theta)$ is increasing.*

Remark 8. Effort maximization in MSS's model is the same as revenue maximization because they assume linear effort costs (and no exclusion). They provide a sufficient condition for full separation in Theorem 4: F satisfies the IFR property.

Infinitely many positional good levels can be implemented by an all-pay auction, in which the more money a consumer pays, the higher status he receives.

Proposition A.2. *The revenue-maximizing mechanism always separates the highest types $\theta \in [\bar{\theta} - \varepsilon, \bar{\theta}]$ for some $\varepsilon > 0$.*

Proof sketch. Because $J'(\bar{\theta}) = 2 > 0$, $J(\theta)$ is increasing at a neighborhood of $\bar{\theta}$, so by Proposition 2 in KMS, types in this neighborhood are separated. □

Remark 9. This "separation at the top" result is a continuous-type analog of MSS's Theorem 3.

Proposition A.3. *The seller can obtain at least half the maximal revenue by offering one positional good level in addition to a free low-level position.*

Proof. The cutoff type indifferent between two levels is

$$\theta^*(p) \frac{1 + F(\theta^*(p))}{2} - p = \theta^*(p) \frac{F(\theta^*(p))}{2} \iff \theta^*(p) = 2p.$$

The revenue, denoted by R_2, is given by

$$R_2 = \max_p p(1 - F(2p)) = \frac{1}{2} \max_p p(1 - F(p)).$$

Because $\mathrm{MPS}(F) \subseteq \mathcal{M}$, we have

$$\max R = \max_{s \in \mathrm{MPS}(F)} \int_0^{\bar{\theta}} J(\theta) s(\theta) \, \mathrm{d}F(\theta) < \max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta) q(\theta) \, \mathrm{d}F(\theta) = \max_p p(1 - F(p)) = 2R_2.$$

The inequality is strict because an extreme point of $\mathcal{M}$ is not in $\mathrm{MPS}(F)$. $\square$

This is a stronger version of Proposition 3. Moreover, Proposition 4 still applies, so the approximation can be arbitrarily close if the type distribution is sufficiently concave.

A.2 Consumer Surplus Maximization

The consumer surplus maximization problem is given by

$$\max_{s(\theta),p(\theta)} \int_0^{\bar{\theta}} U(\theta)\,\mathrm{d}F(\theta) \tag{A.9}$$

subject to constraints (A.2)–(A.6). Consumer surplus can be written as

$$W = \int_0^{\bar{\theta}} \left(\frac{1-F(\theta)}{f(\theta)}\right) s(\theta)\,\mathrm{d}F(\theta) + \alpha\,\mathbf{E}[\theta] + U(0). \tag{A.10}$$

Because $p(0) \geq 0$, $U(0) = -p(0) \geq 0$ (IR) implies $U(0) = p(0) = 0$.

Proposition 2 in KMS immediately implies the following results.

Proposition A.4. *A finer partition of participants increases the consumer surplus if and only if $\frac{1-F(\theta)}{f(\theta)}$ is increasing and decreases the consumer surplus if $\frac{1-F(\theta)}{f(\theta)}$ is decreasing.*

Corollary A.4.1. *The consumer surplus-maximizing mechanism induces*

(i) total pooling if $\frac{1-F(\theta)}{f(\theta)}$ is decreasing (IFR), and

(ii) full separation if and only if $\frac{1-F(\theta)}{f(\theta)}$ is increasing.

Under total pooling, consumer surplus is $(2\alpha+1)\,\mathbf{E}[\theta]/2$.

Corollary A.4.2. *If both $\theta - \frac{1-F(\theta)}{f(\theta)}$ and $\frac{1-F(\theta)}{f(\theta)}$ are increasing (e.g., exponential distribution and Pareto distribution), a finer partition can increase both the seller's revenue and consumer surplus. Thus, full separation maximizes both the revenue and consumer surplus.*

Remark 10. While the results in previous sections are unaffected if the lowest type is changed to $\underline{\theta} > 0$, the consumer surplus-maximizing mechanism (without exclusion) will be affected in that it may involve some pooling at the bottom even if $\frac{1-F(\theta)}{f(\theta)}$ is increasing because pooling at the bottom increases $U(\underline{\theta}) = \underline{\theta}(\underline{s} + \alpha) > 0$ by raising the minimum status $\underline{s} \equiv \min\{s(\theta)\}$.

Proposition A.5. *Total pooling maximizes consumer surplus if and only if*

$$\int_0^\theta \left(\frac{1 - F(x)}{f(x)} - \mathbf{E}[\theta] \right) \mathrm{d}F(x) \geq 0 \textit{ for all } \theta \in [0, \bar{\theta}].$$

Proof. Denote $H(\theta) = \int_0^\theta \frac{1-F(x)}{f(x)} \, \mathrm{d}F(x)$. Then, the condition in the proposition is equivalent to $H(\theta) \geq H(\bar{\theta})F(\theta) = \mathbf{E}[\theta]F(\theta)$ (graphically, $H(\theta)$ lies above the line connecting $H(0) = 0$ and $H(\bar{\theta}) = \mathbf{E}[\theta]$ in the quantile space). Therefore, $H(\bar{\theta})F(\theta)$, which corresponds to total pooling, is the convex hull of H (i.e., the largest convex function that lies below H).[24] By Proposition 2 in KMS, this condition is necessary and sufficient for total pooling to be welfare-maximizing. □

Remark 11. The condition is equivalent to $\mathbf{E}[\tilde{\theta} \mid \tilde{\theta} \geq \theta] - \theta \leq \mathbf{E}[\theta]$ for all $\theta \in [0, \bar{\theta}]$.

Remark 12. The condition is necessary and sufficient for customers' welfare to be higher when one level of positional goods is offered than when *any* $k > 1$ levels are offered. GW's Proposition 1 provides a stronger sufficient condition for customers' welfare to be higher when one level of positional good is offered than when two levels are offered. An even stronger sufficient condition is the IFR property, according to Proposition A.4.

Moreover, for the highest type $\bar{\theta}$, maximizing his utility $U(\bar{\theta})$ subject to constraints (A.2)–(A.6) is equivalent to

$$\max_{s \in \mathrm{MPS}(F)} U(0) + \int_0^{\bar{\theta}} \frac{s(\theta) + \alpha}{f(\theta)} \, \mathrm{d}F(\theta). \tag{A.11}$$

Because $p(0) \geq 0$, (IR) implies $U(0) = p(0) = 0$. By Proposition 2 in KMS, if f is increasing, $s(\theta) = 1/2$ (i.e., total pooling) maximizes $U(\bar{\theta})$. The following proposition shows that it is a sufficient condition for total pooling to maximize $U(\theta)$ for all $\theta \in [0, \bar{\theta}]$ and provides a necessary and sufficient condition.

Proposition A.6. *Total pooling maximizes every consumer's utility if and only if* $F(\theta) \leq \theta / \bar{\theta}$ *(i.e.,* F *first-order stochastic dominates the uniform distribution). A sufficient condition is that* $f(\theta)$ *is increasing.*

Proof sketch. $U(\bar{\theta}) = \int_0^{\bar{\theta}} \frac{s(\theta)}{f(\theta)} \, \mathrm{d}F(\theta) + \alpha\bar{\theta}$. Note that $\int_0^\theta 1/f(x) \, \mathrm{d}F(x) = \theta$, and the condition $F(\theta) \leq \theta/\bar{\theta}$ is equivalent to $\int_0^\theta (1/f(x) - \bar{\theta}) \, \mathrm{d}F(x) \geq 0$. The rest is similar to the proof of Proposition A.5.

Now it suffices to show that $s(\theta) = 1/2$ maximizes $U(\theta)$ for all $\theta \in [0, \bar{\theta}]$ if (and only if, which is trivial) it maximizes $U(\bar{\theta})$. To see this, when $s(\theta) = 1/2$, $U'(\theta) = s(\theta) + \alpha$ is

[24] A piecewise affine function consisting of other points on $H(\theta)$, which corresponds to offering more positional good levels, lies above $H(\bar{\theta})F(\theta)$ and leads to lower consumer surplus.

constant, so $U(\theta)$ is linear, and $U(\theta) = U(\bar{\theta}) \cdot \theta/\bar{\theta}$ for all $\theta \in [0, \bar{\theta}]$. In general, $U(\theta)$ is convex for any $s \in \text{MPS}(F)$, so $U(\theta) \leq U(\bar{\theta}) \cdot \theta/\bar{\theta}$ for all $\theta \in [0, \bar{\theta}]$. □

Remark 13. Proposition A.6 provides the necessary and sufficient condition for all consumers to be worse off after *any* $k > 1$ levels of positional goods are offered than if one level is offered (for free). GW's Proposition 2 shows that if $F(c) \leq c/\bar{c}$, all consumers are worse off after the introduction of one level of priority service.

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