

# Allocating Positional Goods: A Mechanism Design Approach\*

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## Abstract

I study the optimal allocation of positional goods, where consumers' concern for relative consumption generates externalities. Applications include luxury goods, priority services, education, and organizational hierarchies. Using a mechanism design approach, I characterize feasible allocations through a majorization condition. Under Myerson's regularity condition, offering more levels of goods raises the seller's revenue, so the revenue-maximizing mechanism fully separates participants with possible exclusion at the bottom. Offering a single level at a posted price can guarantee at least half the maximum revenue. Holding participation fixed, offering more levels decreases (increases) consumer surplus when the type distribution has an increasing (decreasing) failure rate, while expanding participation always raises surplus. I also characterize the welfare-maximizing mechanism with and without subsidies.

**Keywords:** Positional goods, mechanism design, externalities.

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*“If everyone stands on tiptoe, no one sees better.”*

— Fred Hirsch (1976), *Social Limits to Growth*

## 1 Introduction

Positional goods are goods or services whose value to consumers depends on their relative positions in consumption. Hirsch (1976) introduced the concept of positional good as one that is “either scarce in some absolute or socially imposed sense” or “subject to congestion or crowding through more extensive use” (pp. 27). Because consumers care about relative positions, the allocation of positional goods generates *externalities*. On the one hand, moving up the position of one consumer necessarily pushes down that of another. On the other hand, as more consumers buy the same good, its value depreciates due to more extensive use.

Examples of positional goods are ubiquitous. Many consumer goods, such as jewelry, luxury cars, and non-fungible tokens (NFTs), are positional because individuals derive utility from social comparisons (Frank, 1985a; Carlsson, Johansson-Stenman, and Martinsson, 2007; Lundy et al., 2025). These goods are also known as status goods (Charles, Hurst, and Roussanov, 2009; Rayo, 2013; Bursztyn et al., 2018), as their value stems from the status they confer relative to others. Alternatively, some goods are positional due to capacity constraints. An example is priority services (Gershkov and Winter, 2023), such as priority boarding, priority pick-up or delivery, and express toll roads. Consumers purchase priority services to reduce waiting times by moving ahead of others, but the value diminishes as more consumers purchase it. With multiple priority levels, the value of each level also depends on relative consumption. More broadly, education can be viewed as a positional good: “The value to me of my education depends not only on how much I have but also on how much the man ahead of me in the job line has” (Hirsch, 1976).

Positional goods differ fundamentally from ordinary, nonpositional goods. For ordinary goods, consumers derive utility from the good’s intrinsic quality; for positional goods, consumers’ utility depends on others’ consumption. This distinction gives rise to different optimal mechanisms. When selling ordinary goods of the same intrinsic quality, the optimal mechanism is a simple posted price: the seller sets a uniform price, and each buyer decides whether to purchase. A buyer who purchases the good obtains its value regardless of how many others purchase it. By contrast, for positional goods, the value of the good diminishes as more buyers purchase it: a good meant to confer the highest

status ceases to do so once sold to multiple buyers. Therefore, a posted-price mechanism is no longer optimal, even when goods have the same intrinsic quality. Instead, the seller can extract more revenue by branding some of the goods as a higher tier to price discriminate based on buyers' valuations for status. In practice, sellers often offer multiple tiers of positional goods, and the value of each tier is determined endogenously in equilibrium: Luxury brands range from factory-store items to high-end exclusives, and airlines offer several tiers of priority boarding.

The provision of multiple tiers of positional goods is largely driven by externalities. This logic differs from that in standard price discrimination for ordinary goods (Mussa and Rosen, 1978), where the seller offers products of varying intrinsic quality to screen consumers. In the Mussa-Rosen model, product differentiation hinges on a positive marginal cost of producing a higher-quality good: when the marginal cost is zero, the revenue-maximizing mechanism collapses to offering only the highest-quality good at a posted price. For positional goods, by contrast, the seller's marginal cost of producing a higher tier is virtually zero, since intrinsic quality differs little across tiers, and yet it is still optimal to offer multiple tiers.<sup>1</sup>

The allocation of positional goods has profound welfare implications. While ordinary goods can simply be allocated to all consumers to maximize welfare, a positional good cannot be allocated to all without diminishing its value: "What each of us can achieve, all cannot" (Hirsch, 1976). Consequently, positional goods remain scarce and competitive despite economic growth. More broadly, they induce a zero-sum status game in which agents invest money (or effort) to attain higher relative positions. While the principal can exploit this status game to extract greater revenue, doing so often comes at the expense of agents' welfare, highlighting the need for regulation.<sup>2</sup> Yet regulation is complicated by positional externalities, as consumers may either benefit or suffer from exclusion and increased competition.

In this paper, I study the optimal allocation of positional goods using a mechanism design approach. I assume that buyers care about their relative positions, or *status*, which is determined by the mass of consumers who purchase goods at the same or lower levels. Buyers privately know their types, namely, their valuations of status. The seller offers a

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<sup>1</sup>For pure positional goods, such as credit cards (Bursztyn et al., 2018), NFTs, and priority boarding, tiers differ only in the relative position they confer and cost virtually the same to produce. Even for luxury goods where quality does vary across tiers, the difference is often marginal, and the price premium is predominantly driven by status concerns (Bagwell and Bernheim, 1996; Chao and Schor, 1998; Kapferer and Valette-Florence, 2021).

<sup>2</sup>For example, Asian governments have been orchestrating educational disarmament (The Economist, 2021; Krishna et al., 2026). Luxury consumption has historically been discouraged by various cultural and religious norms.

menu of positional goods with one or more levels and sets a price for each level. Each buyer can purchase a good and attain the status it confers, or opt out and pay nothing.

To derive the optimal mechanism in the presence of positional externalities, I first characterize the set of feasible status allocations—those that can be induced by some allocation of positional goods. Due to these externalities, feasibility imposes a nontrivial constraint: for example, it is impossible to assign every consumer the highest status at its nominal value. Building on the theory of majorization (Kleiner, Moldovanu, and Strack, 2021, henceforth KMS), I show that an increasing interim status allocation is feasible if and only if it is weakly majorized by the buyer's type distribution in quantile space.

Then, I study revenue maximization. I show that, if the type distribution is regular in the sense of Myerson (1981), offering more levels of positional goods increases the seller's revenue. Consequently, the revenue-maximizing mechanism excludes types with negative marginal revenue and fully separates participants. This allocation can be implemented by an all-pay auction with a reserve price, where the buyer who pays more obtains a higher position. In contrast, if Myerson's regularity condition fails, the optimal mechanism pools some types into the same level.

I find that a posted-price mechanism that offers a single good at a fixed price guarantees at least half the maximum revenue. This result is relevant when the number of positional-good levels is subject to regulatory or practical constraints, and it highlights the role of positional externalities. When selling an ordinary good without positional externalities, a posted-price mechanism maximizes revenue. By contrast, when selling positional goods, the seller cannot offer the highest status to all buyers who pay the price without reducing its value. Consequently, the maximum revenue is strictly lower, and a posted-price mechanism is no longer optimal. Nevertheless, selling a single positional good at a posted price guarantees every buyer a status of at least one-half, which suffices to guarantee at least half the maximum revenue. For power and exponential distributions, this approximation ratio exceeds 90%.

I also analyze how the fineness of status allocation and the extent of exclusion affect consumer welfare. These effects are ambiguous *ex ante*: A finer status allocation generates efficiency gains by assigning higher status to consumers who value it more, but also allows the seller to extract more surplus through higher prices. Similarly, expanding coverage (i.e., reducing exclusion) benefits consumers on the extensive margin, but can harm consumers on the intensive margin, as serving more customers can reduce the status of those who are served. I show that, when participation is fixed (for example, when a basic service is available for free), consumer surplus is lower when the status allocation is finer if the distribution of buyer types has an increasing failure rate (IFR).

This result highlights the tension between revenue and consumer surplus under IFR: while offering more levels generates efficiency gains, most are captured by the seller, leaving consumers worse off. As for exclusion, expanding coverage increases consumer surplus, as the gain from additional participants (on the extensive margin) outweighs the loss to existing ones (on the intensive margin). Consequently, regulation that caps the number of levels the seller can offer may or may not benefit consumers, as it leads to coarser status allocation but also higher participation. In contrast, if the type distribution has a decreasing failure rate (DFR), which implies a heavy tail, allowing the seller to offer more levels can increase consumer surplus.

Next, I study optimal mechanisms that maximize consumer surplus. Unlike ordinary goods, a positional good cannot be allocated to all consumers without diminishing its value, so the optimal mechanism may involve exclusion and multiple tiers. When negative transfers are allowed and subject to budget balance, full separation with cross-subsidization and without exclusion maximizes consumer surplus. Intuitively, assortative matching is efficient, and a fixed subsidy can redistribute efficiency gains among consumers while maintaining incentive compatibility (see also [Gershkov and Schweinzer, 2010](#)). This mechanism can be implemented by an all-pay auction with a lump-sum subsidy, where the buyer who pays more obtains a higher position and every buyer receives a fixed subsidy.

By contrast, in applications where the price (i.e., effort) must be nonnegative, such as education and organizational design, I show that total pooling without exclusion maximizes consumer surplus if the distribution of buyer types satisfies IFR. If instead the distribution satisfies DFR (e.g., the Pareto distribution), consumer surplus under the nonnegative price constraint is maximized by full separation without exclusion. Intuitively, while separation harms agents by requiring them to pay more, high types may benefit from it because they value status more, particularly when the distribution has a heavy tail. In both cases, the consumer-optimal mechanism involves no exclusion. Unlike with ordinary goods, exclusion can raise the status of current consumers who would otherwise be pooled with the excluded ones, but its harm to excluded consumers always exceeds its potential benefit.

Finally, I study optimal mechanisms in terms of social welfare, defined as a weighted sum of the seller's revenue and consumer surplus. In the application to education, the welfare weight on revenue captures the productivity of education: the more productive education is relative to pure signaling, the higher the weight. I characterize conditions under which full separation with potential exclusion maximizes social welfare.

The model can also be interpreted as an organizational design problem where a

principal designs the number and size of status categories (Moldovanu, Sela, and Shi, 2007), and agents who care about their status exert effort to climb the hierarchy (or opt out of the status game). There, the “price” of the good is effort, which entails a linear cost, and an agent’s type is their ability, which determines their marginal cost of effort. Under this interpretation, if the ability distribution satisfies Myerson’s regularity condition, the effort-maximizing mechanism excludes the lowest types and fully separates all participating agents. In practice, this maps to “rank-and-yank” systems, which rank employees and terminate those who underperform. For agent welfare, the results imply that an egalitarian organization is optimal under IFR, while a strict hierarchy is optimal under DFR.

Similarly, the results have implications for education, a classic positional good (Hirsch, 1976; Durst, 2021). In many countries, students invest enormous effort (and money) to acquire a better education than their peers, while governments have tried to orchestrate a collective disarmament against education arms races (The Economist, 2021). Consistent with this conventional wisdom, my results show that education arms races hurt students for common ability distributions that have thin tails. In this case, softening competition through admission lotteries or coarser performance rankings increases student welfare.<sup>3</sup> Conversely, when there are a few superstars, meritocracy maximizes average student welfare. Intuitively, although meritocracy creates a rat race that forces students to exert costly effort, higher-ability students may benefit from it because they have lower marginal costs. Thus, when the ability distribution has a thin (heavy) tail, softer competition benefits (harms) students on average. Additionally, from a social perspective, because meritocracy usually increases total effort, the optimal mechanism depends on the productivity of effort in education: if effort is pure signaling, educational disarmament is likely to be optimal; if effort is sufficiently productive, meritocracy is optimal for social welfare. This tradeoff sheds light on why some educational levels (such as graduate schools) use meritocracy while others (such as middle schools) rely on admission lotteries.

As an extension, I show that the main results are robust to alternative specifications of status, including nonlinear (convex or concave) and signaling-based formulations. I also show that the main results extend to the setting where different tiers of positional goods differ in intrinsic quality (à la Mussa and Rosen (1978)). In addition, I study a variant of the model where higher types obtain *lower* net utility from purchasing the good. The results are qualitatively similar but reversed. Finally, speaking to the application

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<sup>3</sup>Krishna et al. (2026) find that pooling a large fraction of the lowest-performing students leads to a Pareto improvement in college admissions.

to priority services, I allow the seller to prolong waiting times by delaying service, and I show that the revenue-maximizing mechanism fully separates buyers with positive virtual valuations and prolongs waiting time of those with negative virtual valuations.

In Appendix A, I consider the case where exclusion is impossible, which applies in many settings where the lowest status is free. For example, a society may not allow exclusion, so investing no effort guarantees the lowest social status (Moldovanu, Sela, and Shi, 2007). Moreover, priority services may be offered alongside free regular services (Gershkov and Winter, 2023). In these cases, the free lowest tier still have positive status value, as the lowest status cannot be offered to multiple agents at its nominal value either. The results are qualitatively similar to the benchmark model with exclusion.

**Literature Review.** The idea that consumers' utility depends on comparisons with others' consumption dates back at least to Veblen (1899) (see also Duesenberry, 1949; Leibenstein, 1950; Bagwell and Bernheim, 1996). Since Hirsch (1976) introduced the concept of positional good, the status interdependence in preference has been formally modeled and studied by Frank (1985b, 2005, 2008), Robson (1992), and Hopkins and Kornienko (2004). This literature focuses on consumer choice in the presence of positional externalities and the welfare effects of changes in income distribution and government policies (e.g., luxury taxes).<sup>4</sup>

Mussa and Rosen (1978) establish the optimality of price discrimination through product differentiation for a monopolist selling ordinary goods. However, as discussed above, the optimal mechanism degenerates to a posted-price mechanism when the marginal cost of producing a higher-quality good is zero. In a more general setting, Anderson and Dana (2009) show that product differentiation can be optimal even when the marginal cost is zero or negative (see also Deneckere and Preston McAfee (1996)), provided consumer utility is strictly log-supermodular—which is violated by the multiplicatively separable specification in Mussa and Rosen. This paper differs from their frameworks in two respects: the value of the good is determined endogenously in equilibrium, and product differentiation is optimal even with zero marginal cost and multiplicatively separable utility.

Moldovanu, Sela, and Shi (2007, henceforth MSS) study the optimal design of the number and size of status categories in an organization to maximize agents' effort when they

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<sup>4</sup>Another strand of literature uses the Duesenberry formulation that stems from Duesenberry (1949) and Pollak (1976) (see the survey by Truyts (2010)), which assumes the consumer's utility depends on the consumption of others in addition to their absolute consumption. Empirical evidence of positional goods and externalities include Luttmer (2005), Alpizar, Carlsson, and Johansson-Stenman (2005), Carlsson, Johansson-Stenman, and Martinsson (2007), Bursztyn et al. (2018).

care about their relative position (i.e., status). They find that full separation is optimal if the ability distribution has an increasing failure rate (IFR), while coarse partitions may be optimal for some distributions—for example, the optimal partition involves only two categories if the distribution is sufficiently concave. Methodologically, I use a mechanism design approach that characterizes feasibility via majorization, while they use optimal contest design with a finite number of agents. Conceptually, my framework generalizes theirs by allowing for *exclusion* of agents and stochastic allocations, while also analyzing agents' welfare.<sup>5</sup> Furthermore, in cases without exclusion, I show that IFR is unnecessary for full separation to maximize effort; in fact, full separation maximizes both effort and welfare if the distribution satisfies Myerson's regularity and has a *decreasing* failure rate.

Relatedly, [Rayo \(2013\)](#) studies a model of monopolist provision of conspicuous goods and characterizes the optimal allocation. His definition of conspicuous goods differs from that of positional goods in that consumers care about the average type in their assigned category instead of relative positions. Despite this difference, his results are qualitatively similar to those of MSS; he also focuses on revenue maximization using deterministic mechanisms without exclusion. In Section 5.1.2, I show that my feasibility condition can be applied to his model of conspicuous goods.<sup>6</sup>

Similar to MSS, [Immorlica, Stoddard, and Syrgkanis \(2015\)](#) study the optimal design of badges to incentivize users' contribution to online communities via status rewards. The most important difference is that they assume status is decreasing in the mass of agents who are *weakly* superior, without adjusting for agents at the same status level. This specification mechanically precludes pooling from being optimal, as it strictly reduces the status of all agents at the same level. Consequently, total pooling would never be optimal—even for maximizing agent welfare—in sharp contrast to my results. Furthermore, exclusion is equivalent to pooling agents at the lowest status tier.

In a different vein, [Gershkov and Winter \(2023, henceforth GW\)](#) study a model of priority service and analyze its implications for consumer welfare. Because priority service (PS) is a positional good arising from capacity constraints, my model extends theirs to multiple (and stochastic) PS levels while allowing for exclusion using a mechanism design approach. By contrast, they focus on the welfare implications of having two PS levels instead of the optimal mechanism.<sup>7</sup>

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<sup>5</sup>Their definition of status is equivalent (up to an affine transformation) to mine (see Footnote 10).

<sup>6</sup>[Board \(2009\)](#) assumes that the value of being in a group category takes a more general form. He shows that, under a regularity condition, the revenue-maximizing mechanism segregates agents too finely and excludes too many agents compared to the socially efficient benchmark. By contrast, the revenue-maximizing mechanism in my model segregates agents (weakly) *less* finely than the socially efficient benchmark, particularly when Myerson's regularity condition fails.

<sup>7</sup>They extend their model to multiple PS levels, but without exclusion, in Section V. In the case without

Another closely related paper is by [Loertscher and Muir \(2022\)](#), who study revenue maximization when selling a fixed inventory of goods with heterogeneous qualities. Because positional externalities can arise from a limited stock, under the linear status specification and without intrinsic value, my model can be viewed as a continuous version of theirs where each quality has a unit stock. In particular, pooling multiple types into the same status corresponds to randomization (or conflation) in their model. Nevertheless, my modeling approach leads to various additional applications, such as luxury goods, education, and organizational status. Moreover, when positional externalities arise from psychological reasons, I consider nonlinear status specifications and varying magnitudes of concern about the number of consumers at the same level.

More broadly, this paper contributes the literature on mechanism design with allocative externalities (e.g., [Jehiel, Moldovanu, and Stacchetti, 1996](#); [Jehiel and Moldovanu, 2006](#); [Ostrizek and Sartori, 2023](#); [Akbarpour, Dworzak, and Kominers, 2024](#); [Dworczak et al., 2024](#)). In particular, this model features a continuum of agents who sort endogenously into different status levels, where positional externalities arise from agents' (endogenous) relative consumption rather than group identities ([Dworczak et al., 2024](#)) or network effects ([Csorba, 2008](#); [Ostrizek and Sartori, 2023](#); [Meisner and Pillath, 2025](#)).

## 2 Model

### 2.1 Setup

A monopolist seller (she) sells positional goods  $x \in X \subseteq \mathbb{R}_{++}$  to a continuum of buyers (he) with unit mass. A larger  $x$  represents a higher-level good, such as a higher-end luxury good or a higher-priority boarding group. Let  $x = 0$  denote the outside option of not buying. Because consumers care about their relative position, consumption of positional goods confers *status* (defined later). Buyers are heterogeneous in their valuations of status (i.e., their types), denoted by  $\theta$ , which has a distribution  $F(\theta)$  with continuous density  $f(\theta)$  and full support on  $\Theta = [0, \bar{\theta}]$ .<sup>8</sup> A buyer with type  $\theta$  receives a payoff of  $u(p, s, \theta) = \theta s - p + v(\theta) \cdot \mathbf{1}_{x>0}$ , where  $s \in [0, 1]$  is the status conferred by the good,  $p$  is the price, and  $v(\theta) \geq 0$  is the intrinsic value of the good. Assume  $v'(\theta) \geq 0$  and  $v''(\theta) \leq 0$ , so that higher types derive greater intrinsic value from the good at a diminishing marginal rate. If the buyer does not receive the good (i.e.,  $x = 0$ ), his status is defined as  $s = 0$ , and

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exclusion, I refine some of their results by providing necessary and sufficient conditions and show some additional results in Appendix A.

<sup>8</sup>The support can be extended to  $\bar{\theta} = \infty$  and  $\underline{\theta} > 0$ . The latter case leads to a complication in consumer surplus maximization under decreasing failure rates (see Remark 5).

he does not obtain the intrinsic value.

**Definition of status.** Let  $G$  denote the distribution of buyers' consumption over  $x \in \tilde{X} \equiv X \cup \{0\}$ , so  $G(x)$  represents the mass of buyers who consume weakly lower-level goods than  $x$  or opt out. Following Robson (1992) and Hopkins and Kornienko (2004), I define the *status* of a buyer who consume  $x \in X$  (i.e., participant) as

$$S(x, G(\cdot)) = \frac{G^-(x) + G(x)}{2} \in [0, 1], \quad (1)$$

where  $G^-(x) \equiv \lim_{x \rightarrow x^-} G(x)$  is the mass of buyers who consume *strictly* lower-level goods than  $x$  (or opt out). In other words, the status equals the mass of buyers who consume strictly lower-level goods plus half the mass of buyers at the same level.<sup>9,10</sup> Buyers who opt out ( $x = 0$ ) have zero status and are ranked strictly below participants (who purchase  $x \in X$ ), so they contribute to the status of participants.

Because buyers only care about the induced status, the seller essentially sells status  $s \in [0, 1]$ , with distribution  $H$ , subject to the feasibility constraint

$$s = \frac{H^-(s) + H(s)}{2} \in [0, 1], \text{ for all } s \in \text{supp}(H) \setminus \{0\}. \quad (2)$$

Agents who do not buy (i.e., excluded agents) receive status  $s = 0$ . For a given level of exclusion, the participants' expected total status is always a constant, regardless of the distribution of status (whether it has mass points due to pooling). This reflects the zero-sum nature of the status game among participants.

This specification of status can arise from interpersonal comparisons (Frank, 1985b; Robson, 1992; Hopkins and Kornienko, 2004), especially for luxury goods. Consumers gain utility from outranking those who buy lower-level goods or cannot afford the good, but gain less from comparisons with consumers who purchase the same level.<sup>11</sup>

In addition to psychological reasons, the specification can also arise from physical

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<sup>9</sup>Hopkins and Kornienko (2004) assume a more general specification:  $S(x, G(\cdot)) = \gamma G(x) + (1-\gamma)G^-(x) + \alpha$ , where  $\gamma \in [0, 1]$  captures the concern about the number of consumers at the same level, and  $\alpha \geq 0$  is a constant representing a guaranteed minimum status for participants. My model incorporates  $\alpha$  through the intrinsic value  $v(\theta)$ , and many results are robust to general  $\gamma$  (see Section 5.1.3).

<sup>10</sup>Without exclusion, an equivalent specification (up to an affine transformation), à la MSS and Dubey and Geanakoplos (2010), is

$$S(x, G(\cdot)) = G^-(x) - (1 - G(x)) \in [-1, 1],$$

so that the expected total status is always zero.

<sup>11</sup>This is equivalent to assuming that they gain utility from ranking above those who purchase strictly lower-level goods or do not purchase and suffer disutility from ranking below those who purchase strictly higher-level goods. See Footnote 10.

constraints. The following example illustrates this through priority services in queuing, where status corresponds to the reduction in waiting time.

**Example 1** (Priority Services (Gershkov and Winter, 2023)). Consider a unit mass of consumers who arrive simultaneously, and a seller serves one consumer per unit time. The seller offers tiered priority services: consumers at higher priority levels are served first, and those at the same level are served in random order. A consumer's payoff is  $u(p, t, \theta) = \tilde{v}(\theta) - \theta t - p$ , where  $t \in [0, 1]$  is the waiting time,  $\theta$  is the cost of waiting, and  $\tilde{v}(\theta)$  is the value of the good. Defining  $s = 1 - t$  as the status (priority value) and  $v(\theta) = \tilde{v}(\theta) - \theta$  as the net value of the good, this model is equivalent to the baseline model.<sup>12</sup> In particular, excluded consumers contribute to participants' status because participants need not wait for those excluded.

More generally, education can also be viewed as a positional good where externalities arise from capacity constraints: an agent's education level determines his position in the job line (or social hierarchy), and his payoff depends on the number of individuals ahead of him or sharing the same position. This perspective connects naturally to the literature on status contests in organizations, where a principal designs status hierarchies to maximize aggregate effort by optimally assigning agents to status categories according to their effort levels.

**Example 2** (Status Contests (Moldovanu, Sela, and Shi, 2007)). Consider a principal who maximizes agents' effort by designing status categories, where agents care about their relative position within the organization. The positional good  $x$  represents the status category, the price  $p \geq 0$  represents the agent's effort, and the type  $\theta$  represents the agent's ability (i.e., the marginal cost of effort). Each agent has a linear effort cost  $p/\theta$  and receives a payoff of  $s - p/\theta$  upon attaining status  $s \in [0, 1]$ . An agent who is excluded receives zero payoff. After scaling agents' payoffs by  $\theta$ , this is a special case of the model with  $v(\theta) = 0$  and  $p \geq 0$ .

## 2.2 A Mechanism Design Approach

Consider a direct mechanism  $(\chi(\theta, \omega), p(\theta))$ , consisting of a payment function  $p: \Theta \rightarrow \mathbb{R}$  and a (potentially stochastic) positional good allocation  $\chi: \Theta \times [0, 1] \rightarrow \tilde{X} = X \cup \{0\}$ , where the random variable  $\omega \in [0, 1]$  captures the randomness in the mechanism. In

<sup>12</sup>The assumption  $v'(\theta) \geq 0$  implies  $\tilde{v}'(\theta) \geq 1$ : higher types have higher net value of the good, even after accounting for waiting costs when served last. Section 5.4 considers an extension in which  $v'(\theta) \leq -1$  (equivalently,  $\tilde{v}'(\theta) \leq 0$ ), so that higher types derive lower net value from the good.

the direct mechanism, the buyer reports his type, and then  $\omega$  is drawn from  $\Omega = [0, 1]$  according to the uniform distribution. The variable  $\omega$  is independent of  $\theta$  and is the same for all buyers. Then, the buyer pays the price  $p(\theta)$  and receives a good  $\chi(\theta, \omega)$ .<sup>13</sup>

Given the allocation  $\chi$ , for any realization  $\omega \in \Omega$ , the distribution of buyers' consumption over  $x \in \tilde{X}$  is given by  $G_\chi(x | \omega) = \int \mathbf{1}[\chi(\theta, \omega) \leq x] dF(\theta)$ . Thus, it is equivalent to consider a direct mechanism  $(s(\theta), p(\theta))$ , where  $s: \Theta \rightarrow [0, 1]$  is the *interim* status induced by the allocation  $\chi$ , given by

$$s(\theta) = \mathbf{E}_\omega[S(\chi(\theta, \omega), G_\chi(\cdot | \omega))].$$

In particular, if  $\chi: \Theta \rightarrow \tilde{X}$  is deterministic, the interim status is given by  $s(\theta) = S(\chi(\theta), G_\chi)$ .

Denote by  $U(\theta) = \theta s(\theta) - p(\theta) + v(\theta)$  the agent's payoff from participating in the mechanism, and normalize the outside option to zero. Thus, a buyer participates if and only if  $U(\theta) \geq 0$ . The following lemmas are implied by standard arguments.

**Lemma 1.** *There exists a cutoff type  $\theta_0 \in [0, \bar{\theta}]$  such that the buyer participates if and only if  $\theta \geq \theta_0$ .*

**Lemma 2.** *A direct mechanism  $(s(\theta), p(\theta))$  is incentive-compatible if and only if*

- $s(\theta)$  is increasing, and
- $U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} (s(t) + v'(t)) dt$  for all  $\theta \in [\theta_0, \bar{\theta}]$ .

An incentive-compatible interim status  $s(\theta)$  is not necessarily feasible, as it may be unable to be induced by an allocation  $\chi$ . Formally, say an interim status  $s(\theta)$  is *feasible* if there exists an allocation  $\chi$  that induces it—i.e.,  $s(\theta) = \mathbf{E}_\omega[S(\chi(\theta, \omega), G_\chi(\cdot | \omega))]$ .

To provide some intuition, consider a deterministic allocation  $\chi: \Theta \rightarrow \tilde{X}$ . Incentive compatibility implies that  $\chi(\theta)$  is increasing because  $s(\theta) = S(\chi(\theta), G_\chi)$  is increasing. When  $\chi(\theta)$  is strictly increasing at  $\theta$ , then  $s(\theta) = F(\theta)$ . Otherwise, when  $\chi(\theta) = x > 0$  is constant on some interval  $[\theta_1, \theta_2]$  (and  $\chi(\theta) \neq x$  outside this interval), then

$$s(\theta) = \frac{G^-(x) + G(x)}{2} = \frac{F(\theta_1) + F(\theta_2)}{2} = \mathbf{E}[F(\tilde{\theta}) | \tilde{\theta} \in [\theta_1, \theta_2]] = \mathbf{E}[F(\tilde{\theta}) | \chi(\tilde{\theta}) = \chi(\theta)]$$

for all  $\theta \in [\theta_1, \theta_2]$ . In either case,  $s(\theta) = S(\chi(\theta), G_\chi) = \mathbf{E}[F(\tilde{\theta}) | \chi(\tilde{\theta}) = \chi(\theta)]$ .

Now I provide the necessary and sufficient condition for an increasing interim status  $s(\theta)$  to be feasible.

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<sup>13</sup>The allocation can be implemented by a pricing scheme as a function of the distribution of positional good levels.

**Theorem 1** (Feasibility). *An increasing  $s$  is feasible if and only if it is weakly majorized by  $F(\theta)$  in quantile space, denoted by  $s \in \text{MPS}_w(F)$ , that is,*

$$\int_t^{\bar{\theta}} s(\theta) dF(\theta) \leq \int_t^{\bar{\theta}} F(\theta) dF(\theta) \text{ for all } t \in [0, \bar{\theta}]. \quad (3)$$

The proof (in Appendix B) is à la Theorem 3 (Border's Theorem) in KMS.

**Remark 1.** For deterministic allocations  $\chi: \Theta \rightarrow \tilde{X}$ ,  $s(\theta)$  must be an extreme point of  $\text{MPS}_w(F)$ —i.e., either the majorization constraint or the monotonicity constraint binds on  $[\theta_0, \bar{\theta}]$  and  $s(\theta) = 0$  for all  $[0, \theta_0]$ . For stochastic allocations  $\chi$ ,  $s(\theta)$  can take other forms.

**Remark 2.** When exclusion is impossible, the equality must hold at  $t = 0$ , that is,  $\mathbf{E}[s] = \mathbf{E}[F] = 1/2$ . Formally,  $s(\theta)$  is a mean-preserving spread of  $F(\theta)$  in quantile space, denoted by  $s \in \text{MPS}(F)$ .<sup>14</sup>

Intuitively, under full separation, the interim status equals the quantile (i.e.,  $s(\theta) = F(\theta)$ ). In contrast, (partially) pooling multiple types into the same positional good makes the status  $s(\theta)$  a mean-preserving spread of  $F(\theta)$  in quantile space because pooling reduces the value of the positional good compared to its original value under full separation.

**Example** (Two Status Levels). Suppose there are two levels of positional goods, low-level  $(p_L, s_L)$  and high-level  $(p_H, s_H)$ , where  $p_L = 0$  (so there is no exclusion). Denote by  $\theta^*$  the cutoff type whose is indifferent between the two (determined by  $p_H$ ). The consumer buys  $s_H$  if and only if  $\theta \geq \theta^*$ . Then, the interim status is

$$s(\theta) = \begin{cases} s_L = F(\theta^*)/2, & \text{if } \theta < \theta^*, \\ s_H = (1 + F(\theta^*))/2, & \text{otherwise.} \end{cases}$$

It is straightforward to check that  $\mathbf{E}[s] = 1/2$  and that  $\int_{\theta}^{\bar{\theta}} s(\theta') dF(\theta') \leq \int_{\theta}^{\bar{\theta}} F(\theta') dF(\theta')$  for all  $\theta \in [0, \bar{\theta}]$  (with equality at  $0, \theta^*$ , and  $\bar{\theta}$ ).

It is worth noting that a binary status allocation  $s(\theta) = \mathbf{1}[\theta \geq \theta^*]$ , which resembles a posted-price mechanism when selling ordinary goods, is infeasible because it is not a mean-preserving spread of  $F(\theta)$  in the quantile space. Intuitively, this is because a good meant to confer the highest status  $s = 1$  ceases to do so once sold to multiple buyers.

<sup>14</sup>Under MSS's specification  $S(x, G) = G^-(x) - (1 - G(x))$  without exclusion, the feasibility condition becomes  $s \in \text{MPS}(2F - 1)$  with  $\mathbf{E}[s] = \mathbf{E}[2F - 1] = 0$ .

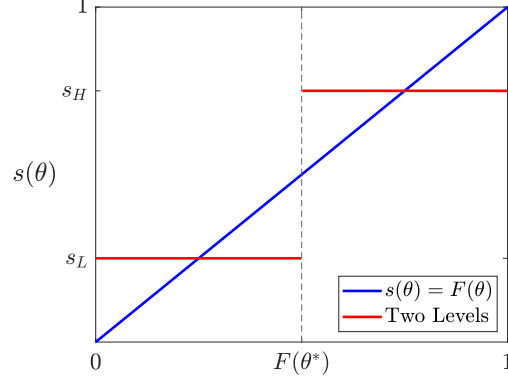


Figure 1: Two Priority Levels

### 3 Revenue Maximization

In this section, I characterize the optimal mechanism that maximizes the seller's revenue and study its welfare implications. Two extreme mechanisms are particularly of interest: full separation (i.e.,  $s(\theta) = F(\theta)$ ) and total pooling among participants (i.e.,  $s(\theta) = (1 + F(\theta_0))/2$ ). The former can be induced by offering a continuum of positional goods that lead to a proliferation of status (i.e., the finest partition), and the latter can be induced by offering a single good.

#### 3.1 Principal's Problem

The revenue maximization problem is given by

$$\max_{s(\theta), p(\theta), \theta_0} \int_{\theta_0}^{\bar{\theta}} p(\theta) dF(\theta) \quad (4)$$

subject to the following constraints for all  $\theta \in [\theta_0, \bar{\theta}]$ :

$$U(\theta) \equiv \theta s(\theta) - p(\theta) + v(\theta) \geq 0 \quad (\text{IR}) \quad (5)$$

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} (s(t) + v'(t)) dt \quad (\text{IC}) \quad (6)$$

$$s(\theta) \text{ is increasing} \quad (7)$$

$$s \in \text{MPS}_w(F) \quad (\text{MPS}) \quad (8)$$

Define  $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$ . Say  $F$  satisfies Myerson's regularity if  $J(\theta)$  is increasing. By

standard arguments, expected revenue is given by

$$R = \int_{\theta_0}^{\bar{\theta}} (J(\theta)s(\theta) - U(\theta_0)) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) \quad (9)$$

By (IR), it is optimal to set  $U(\theta_0) = 0$ .<sup>15</sup> Thus, the revenue maximization problem is equivalent to

$$\max_{s \in \text{MPS}_w(F), \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \quad (10)$$

By KMS, the set  $\text{MPS}_w(F)$  is convex and compact in the norm topology. Thus, the Bauer's maximum principle implies that the maximum is attained at an extreme point of  $\text{MPS}_w(F)$ , which is an extreme point of  $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$  for some  $\theta_0 \in [0, \bar{\theta}]$ , so the optimal mechanism is deterministic.

For any two increasing status allocations  $s, \hat{s} \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$ , say  $s$  is *finer* than  $\hat{s}$  (in terms of majorization) if  $\hat{s} \in \text{MPS}(s)$ —i.e.,  $s \succ \hat{s}$  in the majorization order.<sup>16</sup> By the Fan-Lorentz Theorem (see Theorem 4 in KMS), if  $J(\theta)$  is increasing, then a finer status allocation  $s$  leads to a higher revenue. Hence, we have the following proposition.

**Proposition 2.** *If  $J(\theta)$  is increasing on  $\Theta$ , then*

- (i) *The seller's revenue increases as she offers more levels of positional goods.*<sup>17</sup>
- (ii) *The revenue-maximizing mechanism excludes  $\theta < \theta_0^*$  and fully separates  $\theta \geq \theta_0^*$ , where the optimal cutoff is  $\theta_0^* = \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)F(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0))$ .*

*If  $J(\theta)$  is not monotonic, define  $\tilde{J}(\theta) = \int_0^\theta J(t) dF(t)$  and let  $\text{conv } \tilde{J}$  denote the convex hull of  $\tilde{J}$ . Then, the revenue-maximizing mechanism pools types into the same positional good level if  $\text{conv } \tilde{J}$  is affine and separates types if  $\text{conv } \tilde{J} = \tilde{J}$ .*

The revenue-maximizing mechanism can be implemented by an all-pay auction with a reserve price, where buyers who pay more, conditional on meeting the reserve price, receive higher levels of positional goods. In applications such as education or status within organizations, this may be interpreted as an all-pay contest with a minimum effort requirement.

In general, the optimal mechanism is derived by applying the ironing technique to obtain the convex hull of  $\tilde{J}(\theta)$  (Myerson, 1981; Toikka, 2011). If the virtual valuation  $J(\theta)$

<sup>15</sup>The price is given by  $p(\theta) = v(\theta_0) + \theta_0 s(\theta_0) + \int_{\theta_0}^\theta t ds(t) \geq 0$ .

<sup>16</sup>If  $s$  induces a finer partition of types than  $\hat{s}$ , then  $s$  is *finer* than  $\hat{s}$  (in terms of majorization); the converse is not true.

<sup>17</sup>Formally, let  $R(n)$  denote the seller's maximal revenue when she offers  $n$  levels of positional goods; then,  $R(n)$  is increasing in  $n$ .

is decreasing for some types, the revenue-maximizing mechanism pools them into the same positional good level along with some adjacent types.

The results have several implications. First, under Myerson's regularity condition, it is optimal to offer as many levels of positional goods as possible. As the number of levels increases, the seller can induce a finer status allocation, which not only generates efficiency gains but also allows the seller to extract higher revenue. For example, airlines should introduce more boarding groups or flight classes to the extent regulations and practical constraints allow, and luxury companies should proliferate products from factory stores to high-end exclusives.

Second, compared to selling ordinary goods with intrinsic value  $v(\theta)$  only, attaching status value benefits the seller, which may explain why luxury companies invest heavily in marketing to cultivate status value.

Third, the seller also benefits from excluding low types. In the absence of intrinsic value (i.e., when  $v(\theta) = 0$ ), the revenue-maximizing mechanism excludes all types with negative virtual valuation  $J(\theta) < 0$ . For the uniform distribution on  $[0, 1]$ , exclusion increases revenue by 23.5%. The following corollary characterizes the optimal exclusion level  $\theta_0^*$  in the presence of intrinsic value.

**Corollary 2.1.** *Assume  $F$  satisfies IFR. Then, we have the following:*

- (i) *The optimal exclusion satisfies  $\theta_0^* \leq J^{-1}(0)$ . The equality holds if  $v(\theta)$  is linear in  $\theta$ .*
- (ii) *If  $v(0) \geq (v'(0) + 1)/f(0)$ , then it is optimal to serve every type (i.e.,  $\theta_0^* = 0$ ).*

Under the IFR property, the intrinsic value  $v(\theta)$  reduces the optimal exclusion. When the intrinsic value  $v(0)$  is large enough, and if there are many low types ( $f(0)$  is large), it is optimal for the seller to serve every type.

## 3.2 Approximation

In practice, the number of levels may be constrained by regulation, implementation costs, or other limitations. The following proposition shows that the seller can capture at least half the maximum revenue by selling a single level of good at a posted price.

**Proposition 3.** *The seller can obtain at least half the maximum revenue by selling a single level of good.*

*Proof sketch.* For ease of exposition, assume  $v(\theta) = 0$ , so that the buyer's payoff is  $u(p, s) = \theta s - p$ . Consider an auxiliary problem of selling an indivisible ordinary good, in

which the buyer's payoff is  $u(p, q) = \theta q - p$  when he receives the good with probability  $q \in [0, 1]$ . In this problem, a posted price  $p^* = \arg \max_p p(1 - F(p))$  is optimal, and the optimal allocation is  $q^*(\theta) = \mathbf{1}[\theta \geq p^*]$ . Let  $R_{aux}^* = p^*(1 - F(p^*))$  denote the maximum revenue in the auxiliary problem.

Let  $R^*$  denote the maximum revenue from selling positional goods, and let  $R_1$  denote the maximum revenue from selling a single positional good. When selling positional goods, we have  $R^* < R_{aux}^*$  because the posted-price allocation  $q^*(\theta)$  is infeasible due to externalities. On the other hand, selling a single positional good at price  $\frac{1+F(p^*)}{2}p^*$  also excludes buyers with types  $\theta < p^*$  and guarantees each participant at least  $s = 1/2$ , so the resulting revenue is at least  $\frac{1}{2}R_{aux}^*$ . Hence, we have  $R_1 \geq \frac{1}{2}R_{aux}^* > \frac{1}{2}R^*$ .

The complete proof, which allows for a general intrinsic value  $v(\theta) \geq 0$ , is given in Appendix B.  $\square$

**Remark 3.** The lower bound does not require IFR or Myerson's regularity.

The auxiliary problem of selling indivisible ordinary goods highlights the distinction between selling ordinary goods and positional goods. In the auxiliary problem, selling an item to one consumer has no externalities on others, so the seller can give  $q = 1$  to multiple buyers. By contrast, when selling positional goods, because consumers care about their relative positions, negative externalities arise—the seller cannot give the highest status  $s = 1$  to multiple buyers. Thus, the seller of positional goods is subject to an additional feasibility constraint, resulting in a strictly lower revenue.

**Implications.** When there are costs associated with offering multiple levels of positional goods, the approximation result implies that a finite number of levels is optimal. Formally, when the seller offers  $n \geq 1$  levels of positional goods, let  $R_n$  denote the maximal revenue and  $C_n$  denote the cost, and we have the following result.

**Observation 1.** Assume  $J(\theta)$  is increasing and that  $C_n > 0$  is (weakly) convex in  $n$ . Then, the optimal number of levels  $n^*$  is finite.

The proof is straightforward. By Proposition 2,  $R_n$  is increasing and bounded above by  $R^*$ , so  $R_{n+1} - R_n \rightarrow 0$  as  $n \rightarrow \infty$ . Since  $C_n$  is convex, there exists a finite  $N \geq 1$  such that  $C_{n+1} - C_n > R_{n+1} - R_n$  for all  $n \geq N$ . Therefore,  $\Pi_n := R_n - C_n$  attains its maximum at some finite  $n^* \leq N$ . Furthermore, by Proposition 3, if  $C_2 - C_1 \geq R^*/2$ , then a single level ( $n^* = 1$ ) is optimal.

**Graphical Illustration.** Figure 2 plots the *revenue curve*  $R(\tau) = (1 - \tau)F^{-1}(\tau)$  in the quantile space  $\tau = F(\theta)$ . For simplicity, assume the intrinsic value  $v(\theta) = 0$  for simplicity, as a positive  $v(\theta)$  further increases the approximation ratio  $\frac{R_1}{R^*}$ .

The *blue* area in Figure 2 represents  $R^*$ , the revenue generated by the revenue-maximizing allocation  $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$ . To see this, note that  $R'(\tau) = -J(\tau)$ , so using the change of variable  $\tau = F(\theta)$  (and  $\tau_0^* = F(\theta_0^*)$ ) and integration by parts, the revenue under  $s^*(\theta)$  is

$$\int_{\theta_0^*}^{\bar{\theta}} J(\theta)F(\theta) dF(\theta) = - \int_{\tau_0^*}^1 \tau dR(\tau) = \tau_0^* R(\tau_0^*) + \int_{\tau_0^*}^1 R(\tau) d\tau,$$

which equals the area of the blue region.

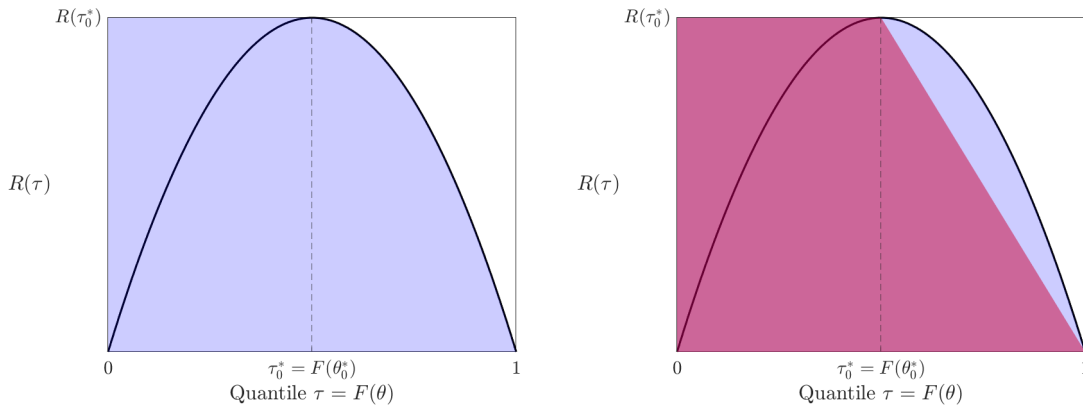


Figure 2: Revenue curve  $R(\tau)$  for the uniform distribution

The *red* trapezoid in the right panel represents the revenue from selling a single good at a posted price  $p_0^* = \frac{1+F(\theta_0^*)}{2}\theta_0^*$ , which induces  $\bar{s}(\theta) = \frac{1+F(\theta_0^*)}{2} \cdot \mathbf{1}[\theta \geq \theta_0^*]$ . This is because the revenue under  $\bar{s}$  is

$$\int_{\theta_0^*}^1 J(\theta) \frac{1 + F(\theta_0^*)}{2} dF(\theta) = \frac{1 + \tau_0^*}{2} R(\tau_0^*),$$

which equals the area of the red trapezoid in the right panel. Since the seller can optimize the price (and setting it at  $p_0^*$  is feasible), the revenue from selling a single good is at least the red area.

Finally, the entire box, with area  $R(\tau_0^*)$ , represents the maximum revenue from selling ordinary goods in the auxiliary problem. It is straightforward to verify that the red area exceeds half of the entire box and is thus strictly larger than half of the blue area.

The approximation is very close for many common distributions, as shown in the

following examples (with  $v(\theta) = 0$ ).

**Example** (Exponential distribution). If  $F(\theta) = 1 - \exp(-\lambda\theta)$  where  $\lambda > 0$ , selling a single good can obtain 91.9% of the maximum revenue.

**Example** (Uniform distribution). If  $F(\theta) = \theta$  on  $[0, 1]$ , selling a single good can obtain 92.4% of the maximum revenue.

**Example** (Power distribution). If  $F(\theta) = \theta^\beta$  on  $[0, 1]$  where  $\beta > 0$ , the approximation ratio is

$$\frac{R_1}{R^*} = \frac{(1 + \beta) \left( \frac{1}{1+2\beta} \right)^{\frac{1}{2\beta}}}{\beta + (1 + \beta)^{-1 - \frac{1}{\beta}}} > 91.4\%.$$

The ratio approaches 1 as  $\beta \rightarrow 0$  (i.e.,  $F$  is sufficiently concave) or  $\beta \rightarrow \infty$  (i.e.,  $F$  is sufficiently convex).

**Example** (Pareto distribution). If  $F(\theta) = 1 - \theta^{-\beta}$  on  $[1, \infty)$  where  $\beta > 1$ , the approximation ratio is

$$\frac{R_1}{R^*} = \left( 1 - \frac{1}{2\beta - 1} \right)^{\frac{\beta-1}{\beta}} > 79.3\%.$$

### 3.3 Welfare Comparative Statics

Although full separation with some exclusion maximizes the seller's revenue, it may or may not benefit consumers. I now analyze how consumer welfare responds to changes in the fineness of status allocation and the extent of exclusion. The goal of this comparative statics analysis is to evaluate consumer welfare when regulations or exogenous restrictions prevent the seller from offering the revenue-maximizing mechanism—for instance, by capping the number of positional-good levels, imposing a price cap on the lowest tier, or requiring the seller to serve all consumer types.

The directions of the welfare effects are ambiguous *ex ante*. On the one hand, a finer status allocation generates efficiency gains by assigning higher status to consumers who value it more, thereby increasing consumer welfare. On the other hand, it also allows the seller to extract more surplus by charging higher prices for higher levels.

Exclusion also creates a tradeoff. On the extensive margin, expanding coverage (i.e., reducing exclusion) always benefits excluded consumers. On the intensive margin, however, expanding coverage reduces the status of those current consumers who are pooled with the additional ones. For example, when more buyers gain access to a luxury good, the status of those who purchase the entry-level good may decline. Likewise, when

a vendor serves more customers, those served at the basic-tier service may experience longer waiting times.

The consumer surplus is given by

$$W = \int_{\theta_0}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\theta_0}^{\bar{\theta}} \left( \frac{1 - F(\theta)}{f(\theta)} (s(\theta) + v'(\theta)) + U(\theta_0) \right) dF(\theta). \quad (11)$$

When the seller sets the price to maximize revenue, the (IR) constraint is binding, which pins down  $U(\theta_0) = 0$ . Hence, the failure rate  $\frac{f(\theta)}{1-F(\theta)}$  plays a crucial role.

Say  $F$  satisfies IFR (DFR) if the failure rate is increasing (decreasing). The following proposition presents comparative statics for the welfare effects of (i) a finer status allocation, and (ii) more consumer participation.

**Proposition 4.** *The following comparative statics hold for consumer surplus.*

(i) *Fixing the exclusion level  $\theta_0$ , a finer status allocation decreases (increases) consumer surplus if  $F$  satisfies IFR (DFR).<sup>18</sup>*

(ii) *Higher participation (i.e., lower exclusion) increases consumer surplus.*

**Remark 4.** Under Myerson's regularity condition (implied by IFR), the revenue-maximizing seller always offers as many levels as possible to separate participants; hence, consumer surplus decreases (increases) as more positional-good levels are available if  $F$  satisfies IFR (DFR).

This proposition highlights the tension between revenue and consumer surplus under IFR: holding participation fixed, as the seller offers more levels of positional goods (and optimally chooses the price and size of each level), revenue increases while consumer surplus decreases. Therefore, under IFR and when exclusion is fixed, the regulator can increase consumer surplus by limiting the number of positional good levels the seller can offer. When there is no exclusion due to a free basic service, I identify the necessary and sufficient condition under which introducing a higher-level good (e.g., a premium service) benefits consumers in Appendix A.2.

Perhaps surprisingly, when the distribution has a *decreasing* failure rate but still satisfies Myerson's regularity condition, revenue and consumer surplus are aligned: both increase as the seller offers more levels of positional goods. Intuitively, high types are more likely to benefit from a finer status allocation than low types because they value

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<sup>18</sup>The exclusion level is fixed when the price of the lowest-level good is fixed. In particular, when there is a free regular service, there is no exclusion (see Appendix A.2).

status more; since a decreasing failure rate implies a heavy-tailed distribution, a finer allocation raises aggregate consumer welfare.

Moreover, expanding coverage increases consumer surplus. While it benefits the additional participants, it also reduces the status of current participants by pooling them with the newcomers. Proposition 4 shows that the gain on the extensive margin outweighs the loss from current participants on the intensive margin.

Finally, a caveat is that if exclusion is *not* held fixed, it becomes ambiguous under IFR whether consumer welfare decreases as more levels of positional goods are available. This is because, as the seller offers more levels, consumer participation also increases, which in turn raises consumer surplus under IFR. The tradeoff depends on the distribution of consumer types; while one can numerically compute the net effect for specific distributions and numbers of levels, there is no simple sufficient condition for its sign. This result highlights the difficulty of regulating the number of positional-good levels: unless the regulator also controls the price of the lowest-level good (for example, when there is a free basic service), it is usually unclear whether such a regulation benefits consumers. Nevertheless, under DFR, consumer surplus unambiguously increases as more levels are offered, even when exclusion is not held fixed, as both effects point in the same direction.

## 4 Welfare Maximization

In this section, I study the optimal mechanism that maximizes welfare. For ordinary goods, the welfare-maximizing mechanism is simply to allocate the good to all consumers. For positional goods, however, the value of the good diminishes as more consumers receive it, so the welfare-maximizing mechanism may involve exclusion and multiple levels of goods.

First, I consider consumer surplus maximization when negative transfers (i.e., subsidies) are allowed and subject to budget balance. Then, I study consumer surplus maximization under the nonnegative price constraint, which is more relevant for applications to education and organizational hierarchies. Finally, I characterize the optimal mechanism that maximizes social welfare, which is the sum of consumer surplus and seller's revenue.

## 4.1 Consumer Surplus

I first consider consumer surplus maximization when negative transfers are allowed and subject to budget balance ( $\mathbf{E}[p(\theta)] = 0$ ). Rewriting consumer surplus as

$$W = \int_{\theta_0}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\theta_0}^{\bar{\theta}} (\theta s(\theta) - p(\theta) + v(\theta)) dF(\theta), \quad (12)$$

it is straightforward to show that consumer surplus is maximized by full separation without exclusion and with cross-subsidization. Intuitively, this is because assortative matching is efficient, and a fixed subsidy can redistribute efficiency gains among consumers while maintaining incentive compatibility.<sup>19</sup>

**Observation 2.** When negative transfers are allowed and subject to budget balance, the consumer surplus-maximizing mechanism is  $s(\theta) = F(\theta)$  and  $p(\theta) = \int_0^\theta t dF(t) - \mathbf{E}[(1 - F(\theta))\theta]$ .

Under this mechanism, higher types pay more to attain higher status than lower types. The payments will be used to cross-subsidize lower types, who accept lower status but are compensated by a subsidy. This mechanism can be implemented through an all-pay auction (without a reserve price) with a lump-sum subsidy of  $\mathbf{E}[(1 - F(\theta))\theta]$ , where buyers who pay more receive higher positions and all buyers receive the subsidy. For sufficiently low types, the transfer  $p(\theta)$  is negative, resulting in a net subsidy.

### 4.1.1 Nonnegative Prices

In many applications, negative transfers (i.e., subsidies) may be infeasible. In particular, in applications to organizational hierarchies and education, the price (i.e., effort) must be nonnegative. Thus, the mechanism described in Observation 2 is infeasible because low types receive a net subsidy ( $p(0) < 0$ ).

Notice that  $W$  (in equation (12)) is decreasing in  $p(\theta)$ , and that  $p(\theta) = p(0) + \int_0^\theta t ds(t)$  is increasing in  $\theta$  by (IC). Hence, under the nonnegative price constraint  $p(\theta) \geq 0$ , consumer surplus is maximized when  $p(0) = 0$  and  $U(0) = v(0)$ . With the payoff of the lowest type pinned down, the comparative statics results (Proposition 4) apply analogously.

**Proposition 5.** *Under the nonnegative price constraint, consumer surplus is maximized by*

- (i) *total pooling without exclusion (i.e.,  $s(\theta) = 1/2$ ) and  $p(\theta) = 0$  if  $F$  satisfies IFR;*

<sup>19</sup>See also Gershkov and Schweinzer (2010). I thank Alex Gershkov for pointing this out.

(ii) full separation without exclusion (i.e.,  $s(\theta) = F(\theta)$ ) and  $p(\theta) = \int_0^\theta t dF(t)$  if  $F$  satisfies DFR.

**Remark 5.** If the lowest type is changed to  $\underline{\theta} > 0$ , then some pooling at the bottom can increase consumer surplus even under DFR, as pooling at the bottom increases  $U(\underline{\theta}) = \underline{\theta}s(\underline{\theta}) + v(\underline{\theta}) > 0$  by raising the lowest status  $s(\underline{\theta})$ .

The result shows that under the nonnegative price constraint, exclusion is detrimental to consumers under both IFR and DFR. Under IFR, because it is optimal to pool participants, exclusion can raise the status of current participants, who would otherwise be pooled with more consumers. Nevertheless, the loss to excluded consumers on the extensive margin exceeds the gain to participants on the intensive margin, so the net effect is detrimental. Under DFR, exclusion has no benefit to current participants because the optimal mechanism does not pool them with additional participants, so no exclusion is optimal.

The result implies that under IFR, the consumer-optimal regulation of positional goods without using subsidies is to restrict the seller to offer a single good and serve all consumers. For example, it is optimal for consumers to have a free regular service.

The results also have implications for organizational hierarchies and education. If the ability distribution has a thin tail, eliminating status competition entirely is optimal for agents' welfare. In the education application, this implies complete educational disarmament—for example, admission lotteries when allocating seats at sought-after schools. If instead there are a few very high abilities (e.g., under the Pareto distribution), meritocracy is optimal for agents' welfare. Intuitively, while meritocracy creates a rat race that forces all agents to exert costly effort, higher-ability agents may benefit from it because of their lower marginal costs. Hence, more (less) intense competition is optimal for agents' welfare when the ability distribution has a heavy (thin) tail.

## 4.2 Social Welfare Maximization

In many settings, a social planner maximizes a weighted sum of the seller's revenue and consumer surplus. Let  $W_S = \mathbb{E}[\lambda p(\theta) + U(\theta)]$  (where  $\lambda \geq 0$ ) denote expected social welfare. In applications to monopolist provision of positional goods, where  $p$  is a transfer,  $\lambda$  is the welfare weight on the seller's revenue relative to consumer surplus. When negative transfers are allowed, I assume  $\lambda \geq 1$ , because otherwise, the social planner would transfer all money from the seller to consumers.

In applications to education or status within organizations, where  $p \geq 0$  is effort,  $\lambda$  captures the degree to which effort is productive from the social perspective. When  $\lambda = 0$ ,

the agents' effort is purely signaling. As  $\lambda$  increases, effort becomes more productive, and the social planner has greater incentives to induce agents to exert effort.

Define the weighted virtual value as  $J_\lambda(\theta) = \lambda\theta - (\lambda - 1)\frac{1-F(\theta)}{f(\theta)}$ . The expected social welfare is

$$W_S = \int_{\theta_0}^{\bar{\theta}} J_\lambda(\theta)s(\theta) dF(\theta) + \int_{\theta_0}^{\bar{\theta}} \left( \lambda v(\theta) - (\lambda - 1)v'(\theta)\frac{1-F(\theta)}{f(\theta)} - (\lambda - 1)U(\theta_0) \right) dF(\theta). \quad (13)$$

Using the same argument as before, we have the following result.

**Proposition 6.** *When negative transfers are allowed and subject to budget balance (and  $\lambda \geq 1$ ), the social welfare-maximizing mechanism induces full separation without exclusion.*

*Under the nonnegative price constraint, if  $J_\lambda(\theta)$  is increasing, the social welfare-maximizing mechanism fully separates the participants and potentially excludes types below a certain threshold.*

In general, the social welfare-maximizing mechanism depends crucially on the welfare weight  $\lambda \geq 0$  on the seller relative to consumers, or the extent to which effort is productive relative to signaling in the application to education.

**Corollary 6.1.** *If  $\lambda = 1$ , the social welfare-maximizing mechanism induces full separation without exclusion.*

Intuitively, when  $\lambda = 1$ , transfers are welfare-neutral. Hence, a finer partition strictly increases social welfare because it increases efficiency by matching the higher types (who value status more) with higher status.

## 5 Extensions

### 5.1 Alternative Specifications of Status

#### 5.1.1 Convex or Concave Status

When positional externalities arise from interpersonal comparison, a concave or convex specification of status can be natural. Formally, assume

$$\tilde{S}(x, G) = \phi(S(x, G)) = \phi\left(\frac{G^-(x) + G(x)}{2}\right), \quad (14)$$

where  $\phi: [0, 1] \rightarrow \mathbb{R}_+$  is continuous and strictly increasing. Let

$$\mathcal{F} = \{\tilde{s} \mid \tilde{s} = \phi \circ s, s \in \text{ext MPS}_w(F)\}$$

denote the set of functions obtained by passing the extreme points of  $\text{MPS}_w(F)$  through  $\phi$ , and let  $\text{conv } \mathcal{F}$  denote the convex hull of the set  $\mathcal{F}$ . Under this specification, an increasing interim status  $\tilde{s}$  is feasible if and only if  $\tilde{s} \in \text{conv } \mathcal{F}$ .

The following proposition shows that the main results are robust to nonlinear status, although stronger regularity conditions may be required. Importantly, as shown in the remark that follows, they also hold when the status specification remains linear, but the agent's payoff is nonlinear in status—i.e.,  $u(p, s, \theta) = \theta\phi(s) - p + v(\theta)$ .

**Proposition 7.** (i) *If  $\phi$  is strictly increasing and convex, Propositions 2, 4, 5, and 6 continue to hold (with a potentially different optimal exclusion level) after replacing the IFR condition with*

$$\frac{1 - F(\theta)}{f(\theta)}\phi'(F(\theta)) \text{ is decreasing.}$$

(ii) *If  $\phi$  is strictly increasing and concave, Propositions 2, 4, 5, and 6 continue to hold (with a potentially different optimal exclusion level) after replacing*

- *Myerson's regularity with*

$$J(\theta)\phi'(F(\theta)) \text{ is increasing;}$$

- *the DFR condition with*

$$\frac{1 - F(\theta)}{f(\theta)}\phi'(F(\theta)) \text{ is increasing;}$$

- *$J_\lambda(\theta)$  is increasing with*

$$J_\lambda(\theta)\phi'(F(\theta)) \text{ is increasing.}$$

**Remark 6.** The results also hold when the status specification remains linear, but the agent's payoff is strictly increasing and convex or concave in status—i.e.,  $u(p, s, \theta) = \theta\phi(s) - p + v(\theta)$ .

In cases where conditions are unchanged (e.g., revenue maximization if  $\phi$  is convex), the result follows from the Fan-Lorentz Theorem. When conditions are stronger (e.g., revenue maximization if  $\phi$  is concave), the proof uses the optimal control method. In par-

ticular, the proposition implies that if  $\phi$  is concave, the revenue-maximizing mechanism may involve randomization even under Myerson’s regularity condition.

### 5.1.2 Signaling

Consider an alternative specification where status arises from signaling. For example, consumers purchase conspicuous goods  $x$  to obtain social status valued at  $\mathbf{E}[\theta \mid x]$  (Rayo, 2013). Under this specification, the status induced by a positional good allocation  $\chi(\theta)$  is given by

$$S(x, G_\chi) = \mathbf{E}[\theta \mid \chi(\theta) = x].$$

An important implication of this specification is that, even if a consumer does not purchase the good (i.e.,  $x = 0$ ), he still obtains a baseline status  $\underline{s} = \mathbf{E}[\theta \mid x = 0]$  rather than zero as in the main specification. Consequently, the buyer’s outside option becomes  $\underline{s}$  instead of zero, and it is never optimal for the seller to exclude any buyer.

Analogous to Theorem 1 (without exclusion), an increasing interim status  $\tilde{s}$  is feasible if and only if  $s \in \text{MPS}(\theta)$  in quantile space. Therefore, the main results continue to hold except that the seller will never exclude any types, analogous to Appendix A where exclusion is impossible. In particular, the revenue-maximizing mechanism fully separates all types if and only if  $J(\theta)$  is increasing, while selling more levels reduces (increases) consumer surplus if  $F$  satisfies the IFR (DFR) property.

### 5.1.3 Varying Positional Concerns about Same-Level Consumers

Now consider a more general specification (Hopkins and Kornienko, 2004)

$$S(x, G(\cdot)) = \gamma G(x) + (1 - \gamma)G^-(x),$$

where  $\gamma \in [0, 1]$  captures the concern about the number of consumers at the same level. The current specification corresponds to  $\gamma = 1/2$ , which ensures that, for a given level of exclusion, the participants’ expected total status is constant regardless of the number of status levels. If  $\gamma < 1/2$  ( $\gamma > 1/2$ ), consumers discount same-level consumers more (less), and pooling decreases (increases) total expected status. The linear case of Immorlica, Stoddard, and Syrgkanis (2015) corresponds to  $\gamma = 0$ , where consumers discount same-level consumers entirely and derive utility from the number of consumers at strictly lower levels. At the other extreme, Frank (1985b) assumes  $\gamma = 1$ , where consumers do not distinguish same-level consumers from lower-level ones.

For revenue maximization, Proposition 2 still holds if  $\gamma \in [0, 1/2]$ . When  $\gamma < 1/2$ , pooling is even less likely to be optimal as it reduces total expected status.

For consumer surplus maximization under nonnegative prices, total pooling without exclusion maximizes consumer surplus under IFR if  $\gamma \in [1/2, 1]$ ; full separation without exclusion maximizes consumer surplus under DFR if  $\gamma \in [0, 1/2]$ . In particular, full separation is always optimal if  $\gamma = 0$  (Immorlica, Stoddard, and Syrgkanis, 2015), while total pooling is always optimal if  $\gamma = 1$  (Frank, 1985b).

## 5.2 Intrinsic Quality

I have maintained the assumption that the positional good has no intrinsic value to abstract from quality differentiation. However, in many applications, a higher-level positional good (e.g., a larger car or higher boarding class) also comes with a higher intrinsic value. Now assume the positional good  $x$  has an intrinsic quality produced at a cost  $c(x)$ , which determines its intrinsic value  $v(x, \theta) = \theta x$ , à la Mussa and Rosen (1978). Assume  $c(x)$  is strictly increasing and strictly convex. Denote by  $x(\theta) = \mathbf{E}_\omega[\chi(\theta, \omega)]$  the interim allocation. Thus, the buyer's utility is  $U(\theta) = \theta(s(\theta) + x(\theta)) - p(\theta)$ .

**Lemma 3.** *A direct mechanism  $(s(\theta), x(\theta), p(\theta))$  is incentive-compatible if and only if*

- $U(\theta) = U(0) + \int_0^\theta (s(t) + x(t)) dt$  for all  $\theta \in [0, \bar{\theta}]$ , and
- $s(\theta) + x(\theta)$  is increasing.

Say  $s$  and  $x$  are *compatible* if  $s(\theta) = \mathbf{E}_\omega[S(\chi(\theta, \omega), G_\chi)]$  and  $x(\theta) = \mathbf{E}_\omega[\chi(\theta, \omega)]$  for some allocation  $\chi: \Theta \times \Omega \rightarrow \tilde{X}$ . Using the same arguments as before, the revenue maximization problem is

$$\max_{s(\theta), x(\theta)} \int_0^{\bar{\theta}} [J(\theta)(s(\theta) + x(\theta)) - c(x(\theta))] dF(\theta) \quad (15)$$

subject to monotonicity of  $s(\theta) + x(\theta)$  and compatibility of  $s$  and  $x$ . Under the regularity condition that  $J(\theta)$  is increasing, we have the following characterization of the revenue-maximizing mechanism.

**Proposition 8.** *Assume  $J(\theta)$  is increasing. The revenue-maximizing mechanism excludes types below  $\theta_0^* = J^{-1}(0)$  and offers  $x^*(\theta) = c'^{-1}(J(\theta))$  to  $\theta \geq \theta_0^*$ .*

The proposition implies that the revenue-maximizing mechanism is robust to the introduction of intrinsic quality. It also has implications for selling positional goods with intrinsic value. By attaching status value, the seller can extract  $\int_{\theta_0^*}^{\bar{\theta}} J(\theta) s^*(\theta) dF(\theta) > 0$

additional revenue. For the uniform distribution on  $[0, 1]$  and quadratic cost  $c(x) = x^2/2$ , revenue with status value is \$0.29 compared to \$0.08 with pure intrinsic value. This may explain why luxury companies invest heavily in cultivating status value.

### 5.3 Delaying Service

Priority service is an example of positional goods that arise from capacity constraints. As shown in Example 1, the baseline model directly applies after defining the status by  $s = 1 - t$ , where  $t$  is the waiting time. In this subsection, I consider an extension where the service provider can delay service and prolong waiting time to  $t = 1 - s > 1$ .

When the seller can delay service up to  $M > 0$  units of time, the range of status allocation  $s$  becomes  $[-M, 1]$ . Incentive compatibility still requires  $s$  to be increasing, and feasibility requires  $s \leq \hat{s}$  for some  $\hat{s} \in \text{MPS}_w(F)$ . Excessive waiting time reduces the buyer's payoff and serves as a substitute for low-type exclusion.

Can the seller benefit from delaying service? Intuitively, instead of excluding low types, the seller can serve them with excessive waiting time, thereby extracting some revenue from them while preventing higher types from mimicking them. Indeed, as shown in the following proposition, it is optimal to assign  $t(\theta) = 1 + v(\theta)/\theta$  (i.e.,  $s(\theta) = -v(\theta)/\theta$ ) to buyers with negative virtual valuation (i.e.,  $J(\theta) < 0$ ). Moreover, under IFR, this increases the mass of consumers who receive zero payoff, effectively increasing exclusion and reducing consumer surplus.

**Proposition 9.** *Assume  $J(\theta)$  is increasing and that delaying service is feasible (with a sufficiently large  $M$ ). Then, the revenue-maximizing mechanism fully separates  $\theta \geq J^{-1}(0)$  and assigns  $t^*(\theta) = 1 + v(\theta)/\theta$  (i.e.,  $s^*(\theta) = -v(\theta)/\theta$ ) to  $\theta < J^{-1}(0)$ .*

*Under IFR, allowing the seller to delay service increases revenue and decreases consumer surplus.<sup>20</sup>*

The allocation  $s^*(\theta) = -v(\theta)/\theta$  is increasing because  $v''(\theta) \leq 0$  and  $v(0) \geq 0$ . It can also be induced by randomization between pooling and separation after pausing the service, as illustrated in the following example.

**Example 3 (Delaying Service).** Consider the payoff  $u(p, s, \theta) = \theta s - p + v(\theta)$  ( $= \tilde{v}(\theta) - \theta t - p$ ), where  $t = 1 - s$  is the waiting time. Assume  $\theta \sim \text{Unif}[0, 1]$  and  $v(\theta) = v_0 + \alpha\theta$ , where

<sup>20</sup>Delaying service strictly increases revenue if  $v$  is nonlinear (i.e.,  $v$  is strictly concave or  $v(0) > 0$ ) and strictly decreases consumer surplus if  $v$  is nonlinear and  $f(0) < \infty$ .

$v_0, \alpha > 0$ . Then, the revenue-maximizing mechanism is

$$t^*(\theta) = \begin{cases} 1 - \theta, & \text{if } \theta \geq 1/2 \\ 1 + \alpha, & \text{if } \theta < 1/2. \end{cases}$$

The excessive waiting time  $1 + \alpha$  can be implemented by pausing service for  $\alpha + 1/4$  units of time before serving types  $\theta < 1/2$  in random order (by pooling them into the same priority level).

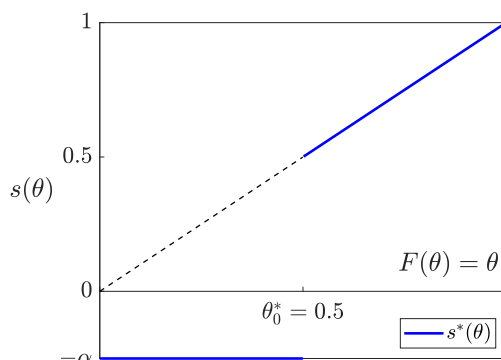


Figure 3: Delaying Service

The insight that excessive waiting time serves as a substitute for exclusion also applies to the case where exclusion is impossible, as the lowest-level good or service is free (see Appendix A). In this case, excessive waiting time diminishes the value of the free service and effectively plays the role of exclusion, thereby increasing the seller's revenue. In particular, by offering a priority service and a free regular service, the seller extracts more revenue as she increases the waiting time for the free regular service.

## 5.4 When Higher Types Obtain Lower Utility

Now I consider the case  $v'(\theta) \leq -1$ , so that higher types obtain lower total utility from purchasing the good, accounting for both status and intrinsic value. This arises naturally in Example 1: if the good has homogeneous value across agents, higher (cost) types derive lower net utility after accounting for waiting costs.<sup>21</sup> Because higher types face higher costs, agents have incentives to *overreport* their types in order to obtain shorter waiting times.

<sup>21</sup>In Example 1, if the good has the same value  $v_0$  to everyone, then the payoff  $u(p, s, \theta) = v_0 - \theta t - p = (v_0 - \theta) + \theta s - p$  (since  $t = 1 - s$ ) satisfies  $v'(\theta) = -1$ .

Formally, since  $U'(\theta) = s(\theta) + v'(\theta) \leq 0$  for all participants when  $v'(\theta) \leq -1$ , higher types have lower payoffs, creating incentives for overreporting—in contrast to underreporting in the main specification. Therefore, it is *high* types that are potentially excluded. Specifically, there exists a cutoff  $\theta_0 \in [0, \bar{\theta}]$  such that a buyer participates (i.e.,  $U(\theta) \geq 0$ ) if and only if  $\theta \leq \theta_0$ . The feasibility condition in terms of majorization continues to hold for all participants  $\theta \in [0, \theta_0]$ .

The following proposition shows the results for the case where  $v'(\theta) \leq -1$ .

**Proposition 10.** *The following results hold if  $v'(\theta) \leq -1$ :*

(i) *Assume  $L(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$  is increasing. Then,*

(a) *The revenue is increasing in the number of positional good levels.*

(b) *The revenue-maximizing mechanism excludes  $\theta > \theta_0^*$  and fully separates  $\theta \leq \theta_0^*$ , where the optimal cutoff is  $\theta_0^* = \arg \max_{\theta_0} \int_0^{\theta_0} L(\theta) F(\theta) dF(\theta) + v(\theta_0) F(\theta_0)$ .*

(c) *If  $-\frac{v'(\theta)}{v(\theta)} \leq \frac{f(\theta)}{F(\theta)}$  for all  $\theta \in [0, \bar{\theta}]$ , then no exclusion is optimal (i.e.,  $\theta_0^* = \bar{\theta}$ ).*

(ii) *The seller can obtain at least half the maximum revenue by selling a single good.*

(iii) *Under the nonnegative price constraint, consumer surplus is maximized by total pooling (full separation) without exclusion if the reverse failure rate  $\frac{f(\theta)}{F(\theta)}$  is decreasing (increasing).*

(iv) *Delaying service cannot increase the seller's revenue.*

Parts (i)–(iii) are qualitatively similar to those in the main specification, with the integrand  $J(\theta)$  replaced by  $L(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$ , and that the revenue-maximizing mechanism excludes high types instead of low types (because  $U'(\theta) \leq 0$ ). For part (iv), when the seller assigns  $t > 1$  (or  $s < 0$ ), she must assign it to low types first because  $s$  is increasing. However, since agents now have incentives to *overreport* (because  $U'(\theta) \leq 0$ ), assigning negative status to *low* types does not mitigate agents' incentives to overreport and therefore cannot increase the seller's revenue.

**Countervailing incentives.** A more complicated case is when  $v'(\theta) \in (-1, 0)$ , so the status game is undesirable for some types and desirable for others. Because  $U'(\theta) = s(\theta) + v'(\theta)$  where  $v'(\theta) \in (-1, 0)$  and  $s: [0, \bar{\theta}] \rightarrow [0, 1]$  is increasing,  $U(\theta)$  is U-shaped, and agents have *countervailing incentives* (see also Lewis and Sappington, 1989): overreporting types when they are low and underreporting types when they are high. Therefore, it is the *intermediate* types that are potentially excluded.

## 6 Conclusion

In this paper, I study the allocation of positional goods using a mechanism design approach. Many goods are positional because consumers care about their relative consumption, either for psychological (such as luxury goods) or physical reasons (such as priority services). Thus, allocating positional goods generates externalities on others: improving one consumer's position necessarily worsens another's, and the value of the good diminishes as more consumers buy it. Because of these externalities, positional goods remain scarce and competitive despite economic growth.

The results show that the seller can extract more revenue by offering many levels of positional goods, exploiting buyers' status concerns. The revenue-maximizing mechanism can be implemented by an all-pay auction with a reserve price, in which buyers who pay more receive higher positions. By contrast, offering more levels is often detrimental to consumers. Thus, the consumer-optimal regulation without subsidies is to restrict the seller to serve a single tier of good to everyone. Nevertheless, when a few buyers have very high valuations, the seller's revenue and consumer surplus both increase with more levels, and the two objectives are aligned.

My model also has implications for the design of status categories within organizations. The results suggest that offering more status levels can increase agents' effort but reduce their welfare, unless there are few superstars with very high abilities, in which case the two objectives are aligned.

The results also shed light on the tradeoffs involved in educational disarmament. Consistent with conventional wisdom, meritocracy usually maximizes students' effort but harms their welfare under common ability distributions that have a thin tail. This supports calls for educational disarmament, such as admission lotteries or coarser performance rankings. Nevertheless, when the distribution has a heavy tail (e.g., under the Pareto distribution), education arms races can be better for students' welfare on average. Thus, the objectives of maximizing effort and welfare can be aligned, challenging the conventional wisdom that meritocracy creates rat races that are detrimental to students. In either case, exclusion always harms students' welfare, even if it can raise the status of participants. Additionally, because meritocracy can raise total effort, when effort is sufficiently productive, educational disarmament also becomes unnecessary.

It remains to be explored how competition among sellers affects the results. This can be interpreted as offering a different track of education leading to different status levels (e.g., vocational versus academic education) or a separate product with its own queue. In either case, consumers of one set of positional goods derive utility independently of those

of the other, either because the goods are psychologically incomparable or physically separate. If the other seller offers the same menu, each seller attracts half the buyers, and such competition can increase consumer surplus. However, the problem of sellers competing strategically in mechanisms remains open.

## Appendix A Optimal Mechanisms without Exclusion

In this section, I study the optimal mechanisms when exclusion is impossible and the lowest-level positional good is free.<sup>22</sup> For example, in [Moldovanu, Sela, and Shi \(2007\)](#), agents cannot be excluded and can get at least the lowest status in the organization; in the benchmark model of [Gershkov and Winter \(2023\)](#), buyers can use the regular (non-priority) service for free. Consequently, the free lowest-level position, rather than opting out, becomes the buyer’s outside option. Moreover, as noted in Remark 2, the feasibility condition becomes  $s \in \text{MPS}(F)$ .

I provide necessary and sufficient conditions under which (i) the revenue-maximizing mechanism is separating or pooling, (ii) consumer welfare is decreasing or increasing in the number of positional good levels, and (iii) introducing a premium tier alongside the free tier increases consumer welfare. I also establish a 2-approximation result: selling a single tier alongside the free tier guarantees at least half the maximum revenue.

### A.1 Revenue Maximization

The revenue maximization problem is given by

$$\max_{s(\theta), p(\theta)} \int_0^{\bar{\theta}} p(\theta) dF(\theta) \quad (\text{A.1})$$

subject to the following constraints on  $\theta \in [0, \bar{\theta}]$

$$U(\theta) - v(\theta) = \theta s(\theta) - p(\theta) \geq \theta \underline{s} \quad (\text{IR}) \quad (\text{A.2})$$

$$U(\theta) - v(\theta) = U(0) - v(0) + \int_0^\theta s(t) dt \quad (\text{IC}) \quad (\text{A.3})$$

$$s(\theta) \text{ is increasing} \quad (\text{A.4})$$

$$s \in \text{MPS}(F) \quad (\text{MPS}) \quad (\text{A.5})$$

where  $\underline{s} = \min\{s(\theta)\}$  denotes the lowest status, which is endogenously determined by the status allocation. The (IR) constraint, which is equivalent to  $\tilde{U}(\theta) \equiv U(\theta) - v(\theta) - \theta \underline{s} \geq 0$ , can be reduced to  $U(0) - v(0) \geq 0$  because  $U'(\theta) - v'(\theta) - \underline{s} = s(\theta) - \underline{s} \geq 0$ .

<sup>22</sup>This section subsumes results in my note “A Mechanism Design Approach to ‘Gainers and Losers in Priority Services.’”

By standard arguments, the expected revenue is given by

$$R = \int_0^{\bar{\theta}} p(\theta) dF(\theta) = \int_0^{\bar{\theta}} \left( \theta - \frac{1 - F(\theta)}{f(\theta)} \right) s(\theta) dF(\theta) - (U(0) - v(0)). \quad (\text{A.6})$$

By (IR), it is optimal to set  $U(0) = v(0)$ . The revenue maximization problem is equivalent to

$$\max_{s \in \text{MPS}(F)} \int_0^{\bar{\theta}} J(\theta) s(\theta) dF(\theta). \quad (\text{A.7})$$

Theorem 4 (Fan-Lorentz) in KMS implies the following result.

**Proposition A.1.** *Offering more positional good levels always (strictly) increases the seller's revenue if and only if  $J(\theta)$  is (strictly) increasing. Thus, full separation maximizes revenue if and only if  $J(\theta)$  is increasing.*

*If  $J(\theta)$  is not monotonic, define  $\tilde{J}(\theta) = \int_0^\theta J(t) dF(t)$  and let  $\text{conv } \tilde{J}$  denote the convex hull of  $\tilde{J}$ . Then, the revenue-maximizing mechanism pools types into the same positional good level if  $\text{conv } \tilde{J}$  is affine and separates types if  $\text{conv } \tilde{J} = \tilde{J}$ .*

**Remark A.1.** GW's Proposition 8 provides a sufficient condition for the seller's revenue to be strictly increasing in the number of priority classes:  $F$  satisfies the IFR property.

**Remark A.2.** Effort maximization in MSS's model is the same as revenue maximization because they assume linear effort costs (and no exclusion). In Theorem 4, they also provide a sufficient condition for full separation :  $F$  satisfies the IFR property.

Infinitely many positional good levels can be implemented by an all-pay auction, in which the more money a consumer pays, the higher status he receives.

**Proposition A.2.** *The revenue-maximizing mechanism always separates the highest types  $\theta \in [\bar{\theta} - \varepsilon, \bar{\theta}]$  for some  $\varepsilon > 0$ .*

*Proof sketch.* Because  $J'(\bar{\theta}) = 2 > 0$ ,  $J(\theta)$  is increasing in a neighborhood of  $\bar{\theta}$ . Moreover,  $J(\bar{\theta}) = \bar{\theta} > J(\theta)$  for all  $\theta < \bar{\theta}$ , Therefore, by Proposition 2 in KMS, types in this neighborhood are separated.  $\square$

**Remark A.3.** This “separation at the top” result is a continuous-type analog of MSS's Theorem 3.

**Proposition A.3.** *The seller can obtain at least half the maximum revenue by selling a single positional good in addition to a free low-level position.*

*Proof.* Denote by  $p$  the price of the high-level position. The cutoff type  $\theta(p)$  indifferent between two levels is given by

$$\theta(p) \frac{1 + F(\theta(p))}{2} - p = \theta(p) \frac{F(\theta(p))}{2} \iff \theta(p) = 2p.$$

Let  $R_2$  denote the seller's maximum revenue from offering one positional good in addition to a free low-level position, which is given by

$$R_2 = \max_p p(1 - F(2p)) = \frac{1}{2} \max_p p(1 - F(p)).$$

Because  $\text{MPS}(F) \subseteq \mathcal{M} \equiv \{x: [0, \bar{\theta}] \rightarrow [0, 1] \mid x \text{ increasing}\}$ , we have

$$\max R = \max_{s \in \text{MPS}(F)} \int_0^{\bar{\theta}} J(\theta) s(\theta) dF(\theta) < \max_{x \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta) x(\theta) dF(\theta) = \max_p p(1 - F(p)) = 2R_2.$$

The inequality is strict because an extreme point of  $\mathcal{M}$  is not in  $\text{MPS}(F)$ . □

This proposition extends Proposition 3 to the case where exclusion is impossible.

**Graphical Illustration.** Figure 4 plots the *revenue curve*  $R(\tau) = (1 - \tau)F^{-1}(\tau)$  in quantile space, where  $\tau = F(\theta)$ . Assume  $v(\theta) = 0$  for simplicity. Similar to Section 3.2, (i) the area under the revenue curve (*blue* area in the left panel) represents the revenue from the revenue-maximizing mechanism—i.e.,  $s^*(\theta) = F(\theta)$ , (ii) the *red* triangle in the right panel represents the revenue from having two levels of positional goods—a higher level offered at price  $F(\theta_0^*)$  in addition to a lower level offered for free, and (iii) the entire box of either panel (which has an area of  $R(\tau_0^*)$ ) represents the maximum revenue from selling nonpositional goods in the auxiliary problem. It is straightforward that the red area (ii) is larger than half of the entire box and thus strictly larger than half of the blue area (i).

The approximation performs well for many common distributions, as shown in the following examples (with  $v(\theta) = 0$ ).

**Example** (Exponential distribution). If  $F(\theta) = 1 - \exp(-\lambda\theta)$  where  $\lambda > 0$ , offering a single good above the free lowest level can obtain 73.6% of the maximum revenue.

**Example** (Uniform distribution). If  $F(\theta) = \theta$  on  $[0, 1]$ , offering a single good above the free lowest level can obtain 75% of the maximum revenue.

**Example** (Power distribution). If  $F(\theta) = \theta^\beta$  on  $[0, 1]$  where  $\beta > 0$ , offering a single good

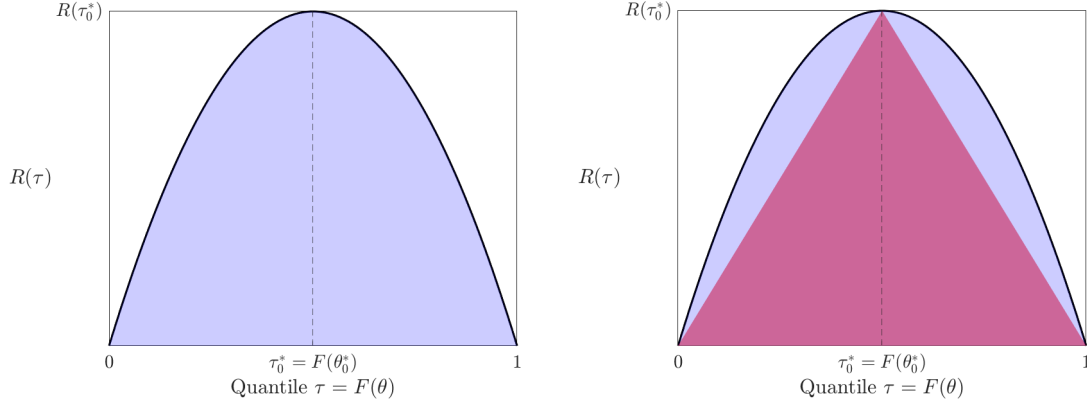


Figure 4: Revenue curve  $R(\tau)$  for the uniform distribution

above the free lowest level can obtain

$$\frac{R_2}{\max R} = \begin{cases} \frac{(1+2\beta)(1+\beta)^{-1/\beta}}{2\beta} > 72.1\%, & \text{if } \beta \geq 1 \\ \frac{\beta(1+2\beta)(1+\beta)^{-1/\beta}}{2\beta^2 - (1+\beta)^{-\frac{\beta+1}{\beta(1-\beta)}} (2\beta^2 + (1+\beta)^{-\frac{\beta}{1-\beta}} - (1+\beta))} > \frac{3}{4}, & \text{if } 0 < \beta < 1 \end{cases}$$

of the maximum revenue. The ratio approaches 1 as  $\beta \rightarrow \infty$  (i.e.,  $F$  is sufficiently convex) or  $\beta \rightarrow 0$  (i.e.,  $F$  is sufficiently concave).

## A.2 Consumer Surplus

Consumer surplus is given by

$$W = \int_0^{\bar{\theta}} U(\theta) dF(\theta) = \int_0^{\bar{\theta}} \left( \frac{1 - F(\theta)}{f(\theta)} \right) s(\theta) dF(\theta) + \mathbf{E}[v(\theta)] + U(0) - v(0). \quad (\text{A.8})$$

**Proposition A.4.** For any increasing allocations  $s, \hat{s}: [0, \bar{\theta}] \rightarrow [0, 1]$  such that  $\hat{s} \in \text{MPS}(s)$ ,

- (i)  $\hat{s}$  results in a higher consumer surplus if  $F$  satisfies IFR;
- (ii)  $\hat{s}$  results in a lower consumer surplus if and only if  $F$  satisfies DFR.

Therefore, consumer surplus decreases (increases) as the seller offers more levels of positional goods if  $F$  satisfies IFR (DFR).

I focus on the case where the price (or effort) is nonnegative, which pin down  $p(0) = v(0) - U(0) = 0$ .

**Corollary A.4.1.** Consumer surplus is the highest when the seller offers

(i) one position level (i.e.,  $s(\theta) = 1/2$ ) if  $F$  satisfies IFR;

(ii) full separation (i.e.,  $s(\theta) = F(\theta)$ ) if and only if  $F$  satisfies DFR.

In the former case, consumer surplus is  $\mathbf{E}[v(\theta) + \theta/2]$ .

**Remark A.4.** GW's Proposition 7 shows the sufficiency of IFR property ( $\frac{1-F(\theta)}{f(\theta)}$  is decreasing) for no priority service to be consumer surplus-maximizing.

**Remark A.5.** If both  $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$  and  $\frac{1-F(\theta)}{f(\theta)}$  are increasing (e.g., exponential and Pareto distributions), a finer partition (in terms of majorization) can increase both the seller's revenue and consumer surplus. Thus, full separation maximizes both the revenue and consumer surplus.

**Proposition A.5.** *Consumer surplus is the highest when the seller offers a single (free) level if and only if*

$$\int_0^\theta \left( \frac{1-F(t)}{f(t)} - \mathbf{E}[\theta] \right) dF(t) \geq 0 \text{ for all } \theta \in [0, \bar{\theta}].$$

*Proof.* Define  $I(\theta) = \int_0^\theta \frac{1-F(t)}{f(t)} dF(t)$ . Then, the condition in the proposition is equivalent to  $I(\theta) \geq I(\bar{\theta})F(\theta) = \mathbf{E}[\theta]F(\theta)$  (i.e.,  $I(\theta)$  lies above the line connecting  $I(0) = 0$  and  $I(\bar{\theta}) = \mathbf{E}[\theta]$  in quantile space). Therefore,  $I(\bar{\theta})F(\theta) = \text{conv } I$  (the largest convex function that lies below  $I$ ). By Proposition 2 in KMS, this condition is necessary and sufficient for total pooling to be welfare-maximizing.  $\square$

**Remark A.6.** The condition is equivalent to  $\mathbf{E}[\tilde{\theta} \mid \tilde{\theta} \geq \theta] - \theta \leq \mathbf{E}[\theta]$  for all  $\theta \in [0, \bar{\theta}]$ .

**Remark A.7.** The condition is necessary and sufficient for customers' welfare to be higher when one level of positional goods is offered (for free) than when *any*  $k > 1$  levels are offered. GW's Proposition 1 provides a stronger sufficient condition for customers' welfare to be higher when one level of positional good is offered than when two levels are offered. An even stronger sufficient condition is the IFR property, according to Proposition A.4.

Moreover, fix a type  $\theta \in [0, \bar{\theta}]$ , maximizing his utility  $U(\theta)$  subject to constraints (A.2)–(A.5) is equivalent to

$$\max_{s \in \text{MPS}(F)} \int_0^\theta \frac{s(t)}{f(t)} dF(t) + v(\theta) \tag{A.9}$$

The following proposition provides a necessary and sufficient condition for total pooling to maximize  $U(\theta)$  for all  $\theta \in [0, \bar{\theta}]$ .

**Proposition A.6.** *Every consumer's utility is the highest when the seller offers one position level if and only if  $F(\theta) \leq \theta/\bar{\theta}$  for all  $\theta \in [0, \bar{\theta}]$  (i.e.,  $F$  first-order stochastically dominates the uniform distribution). A sufficient condition is that  $f(\theta)$  is increasing.*

*Proof sketch.*  $U(\theta) = \int_0^\theta \frac{s(t)}{f(t)} dF(t) + v(\theta)$ . If  $f$  is increasing, then  $\frac{1}{f(\theta)}$  is decreasing, so  $s = 1/2$  maximizes  $U(\theta)$ .

Note that  $\int_0^\theta \frac{1}{f(t)} dF(t) = \theta$ , and the condition  $F(\theta) \leq \theta/\bar{\theta}$  is equivalent to  $\int_0^\theta (\frac{1}{f(t)} - \frac{1}{f(\bar{\theta})}) dF(t) \geq 0$  for all  $\theta \in [0, \bar{\theta}]$ . Similar to the proof of Proposition A.5, one can show that this condition is necessary and sufficient for  $s(\theta) = 1/2$  to maximize  $U(\theta)$  for all  $\theta \in [0, \bar{\theta}]$ .  $\square$

**Remark A.8.** Proposition A.6 provides the necessary and sufficient condition for all consumers to be worse off after *any*  $k > 1$  levels of positional goods are offered than if one level is offered (for free). GW's Proposition 2 shows that if  $F(c) \leq c/\bar{c}$ , all consumers are worse off after the introduction of *one* additional level of priority service.

## Appendix B Proofs

### B.1 Proof of Theorem 1

*Proof.* ( $\implies$ ) follows from  $\mathbf{E}[s(\theta) \mid \theta \geq t] \leq \mathbf{E}[F(\theta) \mid \theta \geq t]$  for all  $t \in [0, \bar{\theta}]$  because switching from any  $s(\theta)$  to assortative rule ( $s(\theta) = F(\theta)$ ) takes the status of low type to high types.

( $\impliedby$ ) According to Corollary 2 in KMS, the extreme points of  $\text{MPS}_w(F)$  are equivalent to the extreme points of  $\text{MPS}(F)$ , which can be implemented by (i) exclusion on  $[0, \theta_0]$  where  $s(\theta) = 0$ ; (ii) pooling on a collection of intervals  $[\underline{\theta}_i, \bar{\theta}_i] \subseteq [\theta_0, \bar{\theta}]$ , where

$$s(\theta) = \frac{1}{F(\bar{\theta}_i) - F(\underline{\theta}_i)} \int_{\underline{\theta}_i}^{\bar{\theta}_i} F(\theta) dF(\theta) = \frac{F(\underline{\theta}_i) + F(\bar{\theta}_i)}{2};$$

and (iii) fully separating otherwise, where  $s(\theta) = F(\theta)$ .

Hence, by Proposition 1 (Choquet's Theorem) in KMS, there exists a probability measure  $\lambda_s$  supported on the extreme points of  $\text{MPS}_w(F)$  such that  $s = \mathbf{E}[\tilde{s} \mid \tilde{s} \sim \lambda_s]$ . Thus,  $s$  can be induced by randomizing over allocations that each induces extreme points of  $\text{MPS}_w(F)$ .  $\square$

## B.2 Proofs of Section 3

### B.2.1 Proofs of Section 3.1

*Proof of Proposition 2.* The following lemma follows from applying Theorem 4 (Fan-Lorentz) in KMS to

$$R(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \left( J(\theta)s(\theta) + v(\theta) - \frac{1 - F(\theta)}{f(\theta)}v'(\theta) \right) dF(\theta). \quad (\text{B.1})$$

**Lemma B.1.** *Assume  $J(\theta)$  is increasing. Fix a cutoff type  $\theta_0$ . For any two increasing allocations  $s, \hat{s}: [\theta_0, \bar{\theta}] \rightarrow [0, 1]$  such that  $\hat{s} \in \text{MPS}(s)$  (i.e.,  $s \succ \hat{s}$ ),  $s$  results in a higher revenue than  $\hat{s}$ .*

Then, part (i) follows from Lemma B.1 because the allocation can be finer (and thus larger in the majorization order) as the number of positional good levels increases. Thus, the revenue-maximizing mechanism fully separates the participants (i.e.,  $s(\theta) = F(\theta)$ ) and the seller's revenue is maximized by the cutoff

$$\begin{aligned} \theta_0^* &= \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} \left( J(\theta)F(\theta) + v(\theta) - \frac{1 - F(\theta)}{f(\theta)}v'(\theta) \right) dF(\theta) \\ &= \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)F(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \end{aligned}$$

If  $J(\theta)$  is not monotonic, the revenue-maximizing mechanism pools types into the same positional good level if  $\text{conv } \tilde{J}$  is affine and separates types if  $\text{conv } \tilde{J} = \tilde{J}$ .  $\square$

*Proof of Corollary 2.1.* Define  $J_v(\theta) = v(\theta) - \frac{1 - F(\theta)}{f(\theta)}v'(\theta)$ . Under IFR,  $J_v(\theta)$  is increasing because  $v(\theta), v'(\theta) \geq 0$  and  $v''(\theta) \leq 0$ . Define

$$\psi(\theta) \equiv -R'(\theta)/f(\theta) = J(\theta)F(\theta) + J_v(\theta). \quad (\text{B.2})$$

Define  $\theta_1 = J^{-1}(0)$ . We have

$$\psi(\theta_1) = J(\theta_1)F(\theta_1) + J_v(\theta_1) = v(\theta_1) - \theta_1 v'(\theta) \geq 0 \quad (\text{B.3})$$

by the concavity of  $v(\theta)$  and  $v(0) = 0$ . Moreover,

$$\psi'(\theta) = J'(\theta)F(\theta) + J(\theta)f(\theta) + J'_v(\theta) > 0 \text{ for all } \theta > \theta_1. \quad (\text{B.4})$$

Thus, for all  $\theta > \theta_1$ , we have  $\psi(\theta) > 0$  and thus  $R'(\theta) < 0$ . Hence, we must have  $\theta_0^* \leq \theta_1$ .

Moreover, because  $J(\theta)$  is increasing,  $J(\theta)F(\theta) > J(0)F(1) = -1/f(0)$ . Thus, if  $J_v(0) - 1/f(0) = v(0) - (v'(0) + 1)/f(0) \geq 0$ , the integrand is always positive, so no exclusion is optimal (i.e.,  $\theta_0^* = 0$ ).  $\square$

*Proof of Proposition 3.* Denote by  $p$  the price of the positional good. The cutoff type  $\theta(p)$  indifferent between buying and not buying is given by

$$\theta(p) \frac{1 + F(\theta(p))}{2} - p + v(\theta(p)) = 0 \implies p = \theta(p) \frac{1 + F(\theta(p))}{2} + v(\theta(p)).$$

Because  $v'(\theta) \geq 0$ , all types above  $\theta(p)$  will buy the good. Thus, by selling a single good, the seller's maximum revenue is

$$R_1 = \max_{\theta} \left( \theta \frac{1 + F(\theta)}{2} + v(\theta) \right) (1 - F(\theta)) \geq \frac{1}{2} \max_{\theta} (\theta + v(\theta)) (1 - F(\theta))$$

because  $\frac{1+F(\theta)}{2}\theta \geq \frac{1}{2}\theta$ . Assume first for simplicity  $v(\theta) = 0$ , so we have  $R_1 \geq \frac{1}{2} \max_{\theta} \theta(1 - F(\theta))$ .

Consider the auxiliary problem of selling an indivisible good to one buyer, in which a standard result implies a posted-price mechanism is optimal (see [Börgers, 2015](#), Proposition 2.5). Formally, denote by  $q$  the probability that the agent receives the good, and let  $\mathcal{M} = \{q: [0, \bar{\theta}] \rightarrow [0, 1] \mid q \text{ increasing}\}$  denote the set of incentive-compatible allocation. The agent's payoff is  $u(p, q, \theta) = \theta q - p$ . Then, the standard result implies that  $\max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) = \max_p p(1 - F(p))$ , and any maximizer  $q^*$  must be an extreme point of  $\mathcal{M}$  (i.e., a posted-price mechanism  $q^*(\theta) = \mathbf{1}[\theta \geq p^*]$ ).

Denote by  $R^*$  the maximum revenue from selling positional goods. Because  $\text{MPS}_w(F) \subseteq \mathcal{M}$ , we have

$$R^* = \max_{s \in \text{MPS}_w(F)} \int_0^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) < \max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) = \max_p p(1 - F(p)) \leq 2R_1.$$

The inequality is strict because any extreme point of  $\mathcal{M}$  is not in  $\text{MPS}_w(F)$  (except when everyone is excluded, which cannot be optimal).

Now consider the general case where  $v(\theta) \geq 0$ . In the auxiliary problem, the agent's payoff becomes  $u(q, p, \theta) = (\theta + v(\theta))q - p$ . We still have

$$\begin{aligned} R^* &= \max_{s \in \text{MPS}_w(F), \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) \\ &< \max_{q \in \mathcal{M}, \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) \end{aligned}$$

$$= \max_{\theta} (\theta + v(\theta))(1 - F(\theta)) \leq 2R_1.$$

In other words, an intrinsic value  $v(\theta) \geq 0$  increases the maximum revenue from selling ordinary goods (which is less than  $2R_1$ ) by at least as much as  $R^*$ , the maximum revenue from selling positional goods.  $\square$

### B.2.2 Proofs of Section 3.3

*Proof of Proposition 4.* (i) The following lemma follows from applying Theorem 4 (Fan-Lorentz) in KMS to

$$W(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \left( \frac{1 - F(\theta)}{f(\theta)} \right) (s(\theta) + v'(\theta)) + U(\theta_0) dF(\theta) \quad (\text{B.5})$$

**Lemma B.2.** *Fix a cutoff type  $\theta_0$ . For any two increasing allocations  $s, \hat{s}: [\theta_0, \bar{\theta}] \rightarrow [0, 1]$  such that  $\hat{s} \in \text{MPS}(s)$ ,  $\hat{s}$  results in a higher (lower) consumer surplus than  $s$  if  $F$  satisfies IFR (DFR).*

Thus, fixing the exclusion level  $\theta_0$ , the consumer surplus decreases (increases) when the status allocation becomes finer under IFR (DFR).

In addition, denote by  $s_k^*$  the revenue-maximizing status allocation chosen by the seller when she offers  $k \geq 1$  levels of positional goods. Under Myerson's regularity (which is implied by IFR), by Lemma B.1,  $s_k^* \in \text{MPS}(s_{k+1}^*)$ . Thus, by Lemma B.2, under IFR (DFR), the consumer surplus is decreasing (increasing) in  $k$ , the number of positional good levels the seller offers.

(ii) Fix any exclusion level  $\theta_0$ . If  $s^*(\theta)$  is *strictly* increasing at  $\theta_0$ , then decreasing  $\theta_0$  to  $\theta_0 - \varepsilon$  increases the consumer surplus for all  $\theta \in [\theta_0 - \varepsilon, \theta_0)$  and does not affect the consumer surplus for all  $\theta \in [\theta_0, \bar{\theta}]$ , so the consumer surplus increases.

If  $s^*(\theta) = s_0$  is constant on  $[\theta_0, \theta_1]$  for some  $\theta_1 \in (\theta_0, \bar{\theta}]$  (and  $s^*(\theta) > s_0$  for  $\theta > \theta_1$ ), then the consumer surplus is

$$W(\theta_0) = \int_{\theta_0}^{\theta_1} \frac{1 - F(\theta)}{f(\theta)} (v'(\theta) + F(\theta_1)/2 + F(\theta_0)/2) dF(\theta) + \int_{\theta_1}^{\bar{\theta}} \frac{1 - F(\theta)}{f(\theta)} (v'(\theta) + s(\theta)) dF(\theta),$$

with derivative given by

$$W'(\theta_0) = -(1 - F(\theta_0)) \left( v'(\theta_0) + \frac{F(\theta_1) + F(\theta_0)}{2} \right) + \frac{f(\theta_0)}{2} \int_{\theta_0}^{\theta_1} (1 - F(\theta)) d\theta \quad (\text{B.6})$$

By the optimality of  $s^*(\theta) = s_0$  is constant on  $[\theta_0, \theta_1]$ , we have

$$\frac{1}{F(\theta) - F(\theta_0)} \int_{\theta_0}^{\theta} \frac{1 - F(t)}{f(t)} dF(t) \geq \frac{1}{F(\theta_1) - F(\theta_0)} \int_{\theta_0}^{\theta_1} \frac{1 - F(t)}{f(t)} dF(t)$$

The inequality holds as  $\theta \rightarrow \theta_0^+$ , so

$$\lim_{\theta \rightarrow \theta_0^+} \frac{\int_{\theta_0}^{\theta} (1 - F(t)) dt}{F(\theta) - F(\theta_0)} = \frac{1 - F(\theta_0)}{f(\theta_0)} \geq \frac{1}{F(\theta_1) - F(\theta_0)} \int_{\theta_0}^{\theta_1} \frac{1 - F(t)}{f(t)} dF(t)$$

Rearranging this yields

$$f(\theta_0) \int_{\theta_0}^{\theta_1} (1 - F(t)) dt \leq (1 - F(\theta_0))(F(\theta_1) - F(\theta_0))$$

Together with equation (B.6), this implies

$$W'(\theta_0) \leq -(1 - F(\theta_0))(v'(\theta_0) + F(\theta_0)) \leq 0.$$

The inequality is strict for all  $\theta \in (0, \bar{\theta})$ . Hence, decreasing  $\theta_0$  increases the consumer surplus  $W(\theta_0)$ .  $\square$

*Proof of Proposition 5.* (i) Under IFR,  $\frac{1-F(\theta)}{f(\theta)}$  is decreasing, so the optimal  $s^*(\theta)$  pools all types  $\theta \in [\theta_0, \bar{\theta}]$ , i.e.,

$$s^*(\theta) = (1 + F(\theta_0))/2 \tag{B.7}$$

Now consumer surplus can be written as a function of the cutoff  $\theta_0 \in [0, \bar{\theta}]$ :

$$W(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \frac{1 - F(\theta)}{f(\theta)} (v'(\theta) + 1/2 + F(\theta_0)/2) dF(\theta) + v(0) \cdot \mathbf{1}[\theta_0 = 0] \tag{B.8}$$

When  $\theta_0 > 0$ , the derivative  $W'(\theta_0)$  is the same as equation (B.6), so the same argument implies that  $W'(\theta_0) \leq 0$  for all  $\theta_0 > 0$  and hence no exclusion is optimal (i.e.,  $\theta_0^* = 0$ ).

(ii) If  $\frac{1-F(\theta)}{f(\theta)}$  is increasing,  $s^*(\theta) = F(\theta)$  for all  $\theta \geq \theta_0$ , so

$$W(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \left( \frac{1 - F(\theta)}{f(\theta)} \right) (F(\theta) + v'(\theta)) dF(\theta) + v(0) \cdot \mathbf{1}[\theta_0 = 0].$$

Therefore, no exclusion is optimal because  $W'(\theta_0) = -(1 - F(\theta_0))(F(\theta_0) + v'(\theta_0)) \leq 0$  for all  $\theta_0 > 0$ .  $\square$

### B.3 Proofs of Section 5

*Proof of Proposition 8.* Because  $J(\theta)$  is increasing, by the rearrangement inequality, it is without loss of optimality to focus on increasing  $s(\theta)$ . The compatibility constraint implies that  $s$  must be feasible, which further implies  $s \in \text{MPS}_w(F)$  because  $s$  is increasing. Ignoring the monotonicity constraint and maximizing  $x$  and  $s$  separately, we have  $x^*(\theta) = c'^{-1}(J(\theta)) \cdot \mathbf{1}[\theta \geq \theta_0^*]$  (by pointwise maximization) and  $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$  (by Proposition 2), where  $\theta_0^* = J^{-1}(0)$ . Because  $s^*(\theta)$  and  $x^*(\theta)$  are increasing in  $\theta$  and can be induced by the same (deterministic) allocation  $\chi^*(\theta, \omega) = x^*(\theta)$ , they are the solution to the original problem.  $\square$

*Proof of Proposition 7.* I prove the result for revenue maximization only; others can be proven analogously.

The revenue maximization problem can be written as

$$\max_{\tilde{s} \in \text{conv } \mathcal{F}, \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) \tilde{s}(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \quad (\text{B.9})$$

Because the objective is linear in  $\tilde{s}$ , it is without loss of optimality to restrict attention to  $\mathcal{F}$  instead of its convex hull. Hence, The maximization problem can be rewritten as

$$\max_{\tilde{s} \in \mathcal{F}, \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) \tilde{s}(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) = \max_{s \in \text{ext MPS}_w(F), \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) \phi(s(\theta)) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \quad (\text{B.10})$$

Now consider a *relaxed* problem where the constraint becomes  $s \in \text{MPS}_w(F)$  (instead of  $s \in \text{ext MPS}_w(F)$ ).

$$\max_{s \in \text{MPS}_w(F), \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) \phi(s(\theta)) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \quad (\text{B.11})$$

The rest of the proof solves the relaxed problem and then show that its solution is an extreme point of  $\text{MPS}_w(F)$ , so it is also the solution to the original problem.

(i) Suppose  $\phi$  is strictly increasing and convex. Then, if  $J(\theta)$  is increasing,  $J(\theta)\phi(s)$  is convex in  $s$  and supermodular in  $(s, \theta)$ . By Fan-Lorentz Theorem, Lemma B.1 still holds (i.e., a finer partition increases the expected revenue). Therefore, the solution to the relaxed problem is  $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$  for some  $\theta_0^*$ . Because  $s^*(\theta)$  is also an extreme point of  $\text{MPS}_w(F)$ , it is also the solution to the original problem.

(ii) Suppose  $\phi$  is strictly increasing and concave. Then, I use the optimal control method to show that  $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$  for some  $\theta_0^*$  if  $J(\theta)\phi'(F(\theta))$  is increasing in  $\theta$ .

Define  $D(\theta) = \int_{\theta}^{\bar{\theta}} (F(t) - s(t)) dF(t) \geq 0$ . The constraints can be written as  $D(\theta) \geq 0$  and  $D'(\theta) = (s(\theta) - F(\theta))f(\theta)$ . Thus, the Hamiltonian is

$$H = J(\theta)\phi(s)f(\theta) + \lambda D + \Lambda(s - F)f, \quad (\text{B.12})$$

where  $\lambda(\theta) \geq 0$  is the Lagrangian multiplier on  $D(\theta) \geq 0$ , and  $\Lambda(\theta)$  is the costate variable associated with  $D'(\theta)$ . The Pontryagin's maximum principle implies that

$$\frac{\partial H}{\partial s} = J(\theta)\phi'(s(\theta))f(\theta) + \Lambda(\theta)f(\theta) = 0, \quad (\text{B.13})$$

$$\frac{\partial H}{\partial D} = \lambda(\theta) = -\dot{\Lambda}(\theta), \quad (\text{B.14})$$

$$\lambda(\theta)D(\theta) = 0, \quad \lambda(\theta) \geq 0, \quad (\text{B.15})$$

$$\Lambda(\theta_0) = 0, \quad (\text{B.16})$$

$$J(\theta_0)\phi(s(\theta_0)) - v'(\theta_0)(1 - F(\theta_0)) + v(\theta_0)f(\theta_0) = 0. \quad (\text{B.17})$$

Note that the problem is concave in  $s$  because  $\phi$  is concave, so the first condition on  $s$  is sufficient for optimality. Therefore, if  $-\Lambda(\theta) = J(\theta)\phi'(F(\theta))$  is increasing in  $\theta$ , the solution to the relaxed problem is  $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$  for some  $\theta_0^*$ . Because  $s^*(\theta)$  is also an extreme point of  $\text{MPS}_w(F)$ , it is also the solution to the original problem.  $\square$

*Proof of Proposition 9.* Denote by  $\text{MPS}_w^-(F)$  by the set of increasing functions  $s: \Theta \rightarrow [-M, 1]$  that satisfies  $s \leq \hat{s}$  for some  $\hat{s}: \Theta \rightarrow [0, 1]$  such that  $\hat{s} \in \text{MPS}_w(F)$ . The revenue maximization problem is

$$\max_{s \in \text{MPS}_w^-(F), \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)).$$

For all  $\theta \geq J^{-1}(0)$ , we have  $s^*(\theta) = F(\theta)$  because  $J(\theta)$  is increasing (and positive). For all  $\theta < J^{-1}(0)$ , the revenue is decreasing in  $s(\theta)$  because  $J(\theta) < 0$ . Because (IR) condition requires  $\theta s(\theta) + v(\theta) \geq 0$ , the optimal  $s^*(\theta) = -v(\theta)/\theta$  is the lowest status that satisfies (IR), which is increasing in  $\theta$  because  $v''(\theta) \leq 0$  and  $v(0) \geq 0$ .  $\square$

*Proof of Proposition 10.* (i) After exchanging the order of integration, the revenue can be written as

$$R = \int_0^{\theta_0} \left( L(\theta)s(\theta) + v(\theta) + \frac{F(\theta)}{f(\theta)}v'(\theta) \right) dF(\theta) \quad (\text{B.18})$$

(ii) The proof is similar to that of Proposition 3. The only difference is that because

$v'(\theta) \leq 0$ , all types  $\theta \leq \theta(p)$  buy the good. Thus, the auxiliary problem becomes a screening problem in procurement where the *lower* price get the good.

(iii) The proof is similar to that of Corollary 5. The only difference is that the consumer surplus is

$$W = \int_0^{\theta_0} \left( -\frac{F(\theta)}{f(\theta)} (s(\theta) + v'(\theta)) \right) + U(\theta_0) dF(\theta). \quad (\text{B.19})$$

Thus, pooling (separating) all participants  $\theta \in [\theta_0, \bar{\theta}]$  is optimal if  $\frac{F(\theta)}{f(\theta)}$  is increasing (decreasing).

In the case of pooling,  $s(\theta_0) = 1 - F(\theta_0)/2$  for all  $\theta \in [0, \theta_0]$ .

$$W(\theta_0) = \int_0^{\theta_0} -\frac{F(\theta)}{f(\theta)} (v'(\theta) + 1 - F(\theta)/2) dF(\theta) + v(\bar{\theta}) \cdot \mathbf{1}_{[\theta_0=\bar{\theta}]}$$

Because  $v'(\theta_0) \leq -1$ , we have

$$W'(\theta_0) = -F(\theta_0)(v'(\theta_0) + 1 - F(\theta_0)/2) + \frac{f(\theta_0)}{2} \int_0^{\theta_0} F(\theta) d\theta \geq 0$$

for all  $\theta_0 < \bar{\theta}$ . Hence, no exclusion is optimal (i.e.,  $\theta_0^* = \bar{\theta}$ ).

In the case of separation, we have  $s(\theta) = F(\theta)$  for all  $\theta \in [0, \theta_0]$ . Thus,  $\theta_0^* = \bar{\theta}$  because exclusion cannot benefit participants (and only harms the excluded buyers). Formally,

$$W(\theta_0) = \int_0^{\theta_0} -\frac{F(\theta)}{f(\theta)} (v'(\theta) + F(\theta)) dF(\theta) + v(\bar{\theta}) \cdot \mathbf{1}_{[\theta_0=\bar{\theta}]}$$

and  $W'(\theta_0) = -F(\theta_0) (v'(\theta_0) + F(\theta_0)) \geq 0$  for all  $\theta_0 < \bar{\theta}$ . □

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