

Allocating Positional Goods: A Mechanism Design Approach*

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Abstract

I study the optimal allocation of positional goods, where consumers' concern for relative consumption generates externalities. Applications include luxury goods, priority services, education, and organizational hierarchies. Using a mechanism design approach, I characterize feasible allocations through a majorization condition. Under Myerson's regularity condition, offering more levels of goods increases revenue; hence, the revenue-maximizing mechanism fully separates participants, with possible exclusion at the bottom. A single level guarantees at least half the maximum revenue. Holding participation fixed, allowing the seller to offer more levels decreases (increases) consumer surplus if the type distribution has an increasing (decreasing) failure rate. When the seller can only offer a single level, expanding coverage may decrease consumer surplus. I also characterize the welfare-maximizing mechanism with and without subsidies.

Keywords: Positional goods, mechanism design, externalities.

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“If everyone stands on tiptoe, no one sees better.”

— Fred Hirsch (1976), *Social Limits to Growth*

1 Introduction

Positional goods are goods or services whose value to consumers depends on their relative position in consumption. Hirsch (1976) introduced the concept of positional goods, describing them as either “scarce in some absolute or socially imposed sense” or “subject to congestion or crowding through more extensive use.” The allocation of such goods generates *externalities*: moving one consumer up necessarily pushes another down, and as more consumers buy the same good, its value to each diminishes.

Examples of positional goods abound. Many consumer goods, such as jewelry, luxury cars, and non-fungible tokens (NFTs), are positional because individuals derive utility from social comparisons (Frank, 1985a; Carlsson, Johansson-Stenman, and Martinsson, 2007; Lundy et al., 2025). These goods are also called status goods (Charles, Hurst, and Roussanov, 2009; Rayo, 2013; Bursztyn et al., 2018), as their value stems from the status they confer relative to others. Other goods are positional due to capacity constraints. An example is priority services (Gershkov and Winter, 2023), such as priority boarding and priority rideshare pick-up. Consumers purchase these services to reduce waiting times by moving ahead of others, but their value diminishes as more consumers purchase them; with multiple priority levels, the value of each level depends on relative consumption as well. More broadly, education can be viewed as a positional good, as students compete for scarce college seats and job opportunities (Durst, 2021; Kim, Tertilt, and Yum, 2024; Krishna et al., 2026).¹

Positional goods differ fundamentally from ordinary goods. For ordinary goods, consumers derive utility from the good’s intrinsic quality; for positional goods, consumers’ utility depends on others’ consumption. This distinction gives rise to different optimal mechanisms. When selling ordinary goods of the same intrinsic quality, the optimal mechanism is a simple posted price: the seller sets a uniform price, and each buyer decides whether to purchase. A buyer who purchases the good obtains its value regardless of how many others do so. By contrast, for positional goods, the value of the good diminishes as more buyers purchase it: a good meant to confer the highest status ceases

¹Education is the classic example in Hirsch (1976): “The value to me of my education depends not only on how much I have but also on how much the man ahead of me in the job line has.” In East Asia, the education system is viewed as positional (The Economist, 2021) and “a zero-sum game” (Jia, Li, and Cousineau, 2025).

to do so once sold to many buyers. Therefore, a posted-price mechanism is no longer optimal, even when goods have the same intrinsic quality. Instead, the seller can extract more revenue by branding some of the goods as a higher tier to price discriminate based on buyers' valuations for status. In practice, sellers often offer multiple tiers of positional goods whose values are determined endogenously in equilibrium: luxury brands range from factory-store items to high-end exclusives, and airlines offer several tiers of priority boarding.

The provision of multiple tiers of positional goods is largely driven by externalities. This logic differs from that in standard second-degree price discrimination for ordinary goods (Mussa and Rosen, 1978), where the seller offers products of varying intrinsic quality to screen consumers. There, when the buyer's utility is multiplicatively separable, product differentiation hinges on a positive marginal cost of producing a higher-quality good: when the marginal cost is zero, the revenue-maximizing mechanism collapses to a posted price for the highest-quality good.² For positional goods, by contrast, the marginal cost of producing a higher tier is virtually zero because intrinsic quality differs little across tiers, and yet offering multiple tiers remains optimal.³

Positional goods also raise distinct welfare considerations. Unlike ordinary goods, which can in principle be allocated to all consumers, a positional good cannot be allocated to all without diminishing its value: "what each of us can achieve, all cannot" (Hirsch, 1976). As a result, positional goods remain scarce and competitive despite economic growth. This raises a natural question: how should such goods be allocated from a welfare perspective? A common view is that positional goods create a zero-sum status game, making them harmful to consumers and providing a rationale for regulation that dampens status competition. Yet positional externalities make the welfare effects ambiguous: softening competition can benefit some consumers while harming others. Similarly, expanding access involves a tradeoff between the welfare gains to newly served consumers and the losses to existing ones whose status is diluted.

In this paper, I study the optimal allocation of positional goods using a mechanism design approach. I assume that buyers care about their relative positions, or *status*, defined by the mass of consumers who purchase goods at lower levels (or opt out) plus

²In the original Mussa–Rosen model where the buyer's utility is linear in quality, product differentiation further requires a strictly increasing marginal cost: with a constant marginal cost, the optimal mechanism still collapses to a posted price.

³For pure positional goods, such as credit cards (Bursztyn et al., 2018), NFTs, and priority boarding, tiers differ only in the relative position they confer and cost virtually the same to produce. Even for luxury goods where quality does vary across tiers, the difference is often marginal, and the price premium is predominantly driven by status concerns (Bagwell and Bernheim, 1996; Chao and Schor, 1998; Kapferer and Valette-Florence, 2021).

half the mass of consumers at the same level. Buyers privately know their valuations of status (i.e., types). The seller offers a menu of positional goods with one or more levels and sets a price for each level. Each buyer can purchase a good to obtain its intrinsic value and the status it confers, or opt out and pay nothing. Using a mechanism design approach, I study direct mechanisms that specify a status profile and a payment schedule as functions of buyers' (reported) types.

I first characterize which status profiles are feasible, in the sense that they can arise from some allocation of positional goods. Positional externalities impose a nontrivial feasibility constraint: for example, it is impossible to assign every consumer the highest status at its nominal value. Building on the theory of extreme points and majorization (Kleiner, Moldovanu, and Strack, 2021, henceforth KMS), I show that feasibility admits a simple characterization in quantile space. After transforming types into quantile ranks, an incentive-compatible status profile is feasible if and only if it is a mean-preserving spread of (i.e., majorized by) the identity function on the participating interval. Intuitively, if every participating buyer receives a distinct tier, they are fully separated, and each participant's status is their quantile rank. At the other extreme, if all participants are pooled into a single tier, their status is the average quantile rank of the participating interval. In general, pooling buyers into the same tier coarsens the status profile while preserving average status, making it a mean-preserving spread of the identity function.

Then, I turn to revenue maximization. If the type distribution is regular in the sense of Myerson (1981), the seller's revenue increases as she offers more levels of positional goods. Therefore, the revenue-maximizing mechanism fully separates participants with possible exclusion at the bottom. This allocation can be implemented by an all-pay auction with a reserve price, where the buyer who pays more obtains a higher position. If the type distribution violates regularity, the optimal mechanism pools some types into the same level.

I show that a posted-price mechanism offering a single good at a fixed price guarantees at least half the maximum revenue. This result is relevant when the number of positional-good levels is subject to regulatory or practical constraints. It also highlights the role of positional externalities. When selling an ordinary good without positional externalities, a posted-price mechanism maximizes revenue. By contrast, when selling positional goods, the seller cannot offer the highest status to all buyers who pay for it without reducing its value. Consequently, the maximum revenue is strictly lower, and a posted-price mechanism is no longer optimal. Nevertheless, selling a single positional good at a posted price guarantees every participant a status of at least one-half, which suffices to guarantee at least half the maximum revenue.

I also analyze the welfare effects of changes in the number of levels and service coverage. The directions of these effects are *a priori* ambiguous. A finer status profile generates efficiency gains by assigning higher status to consumers who value it more, but also allows the seller to extract more surplus through higher prices. When participation is fixed (for example, when a basic tier is available for free), I show that consumer surplus is lower when the seller is allowed to offer more levels if the type distribution has an increasing failure rate (IFR). While the seller creates efficiency gains by offering more levels, she extracts more surplus than she generates, leaving consumers worse off. If, instead, the distribution has a decreasing failure rate (DFR), allowing the seller to offer more levels increases consumer surplus. Under DFR, superstars at the top benefit from a finer status; although low types still lose, the heavy tail places enough mass on these superstars that their gains dominate.

Similarly, expanding coverage (by lowering the price of the lowest tier) has mixed effects on welfare: it benefits consumers on the extensive margin but can be harmful on the intensive margin, as serving more customers may reduce the status of existing ones. When the number of levels is unconstrained, expanding coverage always increases consumer surplus under Myerson's regularity condition, because existing participants are not pooled with new entrants and thus do not lose status. However, when the seller is restricted to a single tier, expanding coverage may decrease or increase consumer surplus. Under IFR, though, the gain to new participants outweighs the loss to existing ones, so consumer surplus rises.

Next, I study optimal mechanisms that maximize consumer surplus. Unlike ordinary goods, a positional good cannot be allocated to all consumers without diminishing its value, so the optimal mechanism may involve exclusion and multiple tiers. When negative transfers are allowed and subject to budget balance, full separation with cross-subsidization and without exclusion maximizes consumer surplus. Intuitively, assortative matching is efficient, and a fixed subsidy can redistribute efficiency gains among consumers while maintaining incentive compatibility (see also [Gershkov and Schweinzer, 2010](#)). This mechanism can be implemented by an all-pay auction with a lump-sum subsidy financed by the auction payments, where the buyer who pays more obtains a higher position and every buyer receives the same rebate.

When subsidies are unavailable, total pooling without exclusion maximizes consumer surplus if the type distribution satisfies IFR. If instead the distribution satisfies DFR (e.g., Pareto), consumer surplus under the nonnegative-price constraint is maximized by full separation without exclusion. Intuitively, separation harms agents by requiring them to pay more, but high types may benefit because they value status more, particularly

when the distribution has a heavy tail. In both cases, the consumer-optimal mechanism involves no exclusion. Unlike with ordinary goods, exclusion can raise the status of existing consumers who would otherwise be pooled with the excluded, but its harm to excluded consumers always exceeds its potential benefit.

Finally, I study social welfare, defined as a weighted sum of the seller's revenue and consumer surplus. In the application to education, the welfare weight on revenue captures the productivity of education: the higher the weight, the more productive education is relative to pure signaling. I characterize conditions under which full separation with potential exclusion maximizes social welfare.

The model can also be interpreted as an organizational design problem in which a principal designs the number and size of status categories (Moldovanu, Sela, and Shi, 2007), and agents who care about their status exert effort to climb the hierarchy or opt out. Under this interpretation, the "price" is effort, which is nonnegative and entails a linear cost, and an agent's type is their ability, which determines the marginal cost of effort. If the ability distribution satisfies Myerson's regularity condition, the effort-maximizing mechanism excludes the lowest types and fully separates all participating agents. This resembles the "rank-and-yank" system, which ranks employees and terminates underperformers. In terms of agent welfare, the results imply that an egalitarian organization is optimal under IFR, while a strict hierarchy is optimal under DFR.

The results also have implications for education. In many countries, students invest enormous effort and money to acquire a better education than their peers, even as governments attempt to orchestrate collective disarmament against education arms races (The Economist, 2021). Consistent with this conventional wisdom, my results show that education arms races hurt students for common ability distributions with thin tails. In this case, softening competition through admission lotteries or coarser performance rankings increases student welfare.⁴ Conversely, when there are a few superstars in the upper tail, meritocracy maximizes aggregate student welfare. While meritocracy creates a rat race that forces students to exert effort, superstars can benefit from it because they have lower marginal costs and are sufficiently spread out in the tail. Although low-ability agents still suffer, the heavy tail also makes the gain to higher-ability agents dominate in the aggregate. Thus, when the ability distribution has a thin (heavy) tail, softer competition benefits (harms) students on average. This may explain why admissions at higher levels of education, such as graduate schools, are more meritocratic, while those at lower levels rely on lotteries.

⁴Krishna et al. (2026) find that pooling a large fraction of the lowest-performing students leads to a Pareto improvement in college admissions.

As an extension, I show that the main results are robust to alternative specifications of status, including convex or concave specifications and signaling-based formulations. The revenue maximization result also extends to the setting where different tiers of positional goods differ in intrinsic quality (à la [Mussa and Rosen \(1978\)](#)). In addition, motivated by the priority services application, I allow the seller to delay service and use excessive waiting time as an ordeal mechanism to screen consumers. Instead of excluding types with negative virtual valuations, the revenue-maximizing mechanism serves them with excessive waiting time. Finally, I study a variant of the model where higher types obtain *lower* net utility from purchasing the good; the results are qualitatively similar, but the relevant conditions are reversed, as agents now have incentives to overreport.

In [Appendix A](#), I consider the case where exclusion is impossible, as in many settings where the lowest status is free. A society may not allow exclusion, so investing no effort guarantees the lowest social status ([Moldovanu, Sela, and Shi, 2007](#)); similarly, priority services may be offered alongside free regular services ([Gershkov and Winter, 2023](#)). In these cases, the free lowest tier still has positive status value, since the lowest status cannot be offered to multiple agents at its nominal value either. The results are qualitatively similar to the benchmark model with exclusion.

Literature Review. The idea that consumers' utility depends on comparisons with others' consumption dates back at least to [Veblen \(1899\)](#) (see also [Duesenberry, 1949](#); [Leibenstein, 1950](#); [Bagwell and Bernheim, 1996](#)). The theoretical literature on positional goods focuses on consumer choice in purchasing positional goods and the welfare effects of income distribution and government policies ([Frank, 1985b, 2005, 2008](#); [Robson, 1992](#); [Hopkins and Kornienko, 2004](#)).⁵ This paper complements the literature by studying the supply side: the monopolist provision of positional goods.

The monopolist provision of nonpositional goods has been studied extensively. [Mussa and Rosen \(1978\)](#) establish the optimality of price discrimination through product differentiation, but the optimal mechanism degenerates to a posted price when the marginal cost is constant. More generally, with multiplicatively separable utility, a posted price is optimal when the marginal cost is zero. When multiplicative separability is relaxed, [Anderson and Dana \(2009\)](#) show that product differentiation can be optimal when the marginal cost is zero or negative (see also [Deneckere and McAfee \(1996\)](#)). This paper

⁵Another strand of literature uses the formulation that stems from [Duesenberry \(1949\)](#) and [Pollak \(1976\)](#) (see the survey by [Truys \(2010\)](#)), which assumes the consumer's utility depends on others' consumption in addition to their absolute consumption. Empirical evidence of positional goods and externalities includes [Luttmer \(2005\)](#), [Alpizar, Carlsson, and Johansson-Stenman \(2005\)](#), [Carlsson, Johansson-Stenman, and Martinsson \(2007\)](#), [Burszty et al. \(2018\)](#).

differs from both frameworks in two respects: the value of positional goods is determined endogenously in equilibrium, and product differentiation is optimal even with zero marginal cost and multiplicatively separable utility.

Moldovanu, Sela, and Shi (2007, henceforth MSS) study the optimal number and size of status categories in an organization to maximize agents' effort when agents care about their relative position. They find that full separation is optimal if the ability distribution has an increasing failure rate (IFR). Methodologically, I use a mechanism design approach that characterizes feasibility via majorization, while they use optimal contest design with a finite number of agents. Conceptually, my framework generalizes theirs by allowing for *exclusion* of agents and stochastic allocations, while also analyzing agents' welfare.⁶ Furthermore, in cases without exclusion, I show that IFR is unnecessary for full separation to maximize effort; in fact, full separation maximizes both effort and welfare if the distribution satisfies Myerson's regularity and has a *decreasing* failure rate.

Relatedly, Rayo (2013) studies monopolist provision of conspicuous goods and characterizes the optimal allocation. Although his definition of conspicuous goods is different in that consumers care about the average type in their assigned category rather than relative positions, his results are qualitatively similar to those of MSS. In Section 5.1.2, I show an analogous feasibility condition for his specification.⁷

Similar to MSS, Immorlica, Stoddard, and Syrgkanis (2015) study the optimal design of badges to incentivize contribution to online communities via status rewards. The most important difference is that they assume status is decreasing in the mass of agents who are *weakly* superior, including those at the same level. Under this specification, pooling assigns agents in the pool the status of the lowest type of that pool, thereby lowering status for all other pooled agents relative to full separation. Consequently, pooling would not arise in a consumer welfare maximization. Moreover, under their specification, opting out is equivalent to pooling at the bottom tier.

In a different vein, Gershkov and Winter (2023, henceforth GW) study priority service and analyze its welfare implications. Because priority service is a positional good arising from capacity constraints, my results extend theirs to multiple (and stochastic) priority levels while allowing for exclusion using a mechanism design approach. By contrast, they focus on the welfare implications of offering two priority levels.⁸

⁶Their definition of status is equivalent, up to an affine transformation, to mine (see Footnote 12).

⁷In addition, Board (2009) allows the value of belonging to a category to take a more general form. He shows that, under a regularity condition, the revenue-maximizing mechanism segregates agents too finely and excludes too many agents compared to the socially efficient benchmark. By contrast, the revenue-maximizing mechanism in my model segregates agents (weakly) *less* finely than the efficient benchmark, particularly when Myerson's regularity condition fails.

⁸They extend their model to multiple priority levels without exclusion in their Section V. For that case,

Another closely related paper is by [Loertscher and Muir \(2022\)](#), who study revenue maximization when selling a fixed inventory of goods with heterogeneous qualities. Because positional externalities can arise from limited stock, under the linear status specification and without intrinsic value, my model can be viewed as a continuous version of theirs in which each quality has a unit stock. In particular, pooling multiple types into the same status corresponds to randomization (or conflation) in their model. My modeling approach, however, accommodates additional applications, such as luxury goods, education, and organizational status, where positional externalities arise from psychological rather than physical scarcity. I also consider nonlinear status specifications and varying magnitudes of concern about the number of consumers at the same level.

More broadly, this paper contributes to the literature on mechanism design with allocative externalities (e.g., [Jehiel, Moldovanu, and Stacchetti, 1996](#); [Jehiel and Moldovanu, 2006](#); [Ostrizek and Sartori, 2023](#); [Akbarpour, Dworzak, and Kominers, 2024](#); [Dworczak et al., 2026](#)). The model features a continuum of agents who sort endogenously into tiers that induce different status levels; positional externalities arise from agents' relative consumption rather than from average characteristics of others ([Dworczak et al., 2026](#)) or network effects ([Csorba, 2008](#); [Ostrizek and Sartori, 2023](#); [Meisner and Pillath, 2025](#)).

Finally, this paper relates to the literature on signaling and matching tournaments (e.g., [McAfee, 2002](#); [Hoppe, Moldovanu, and Sela, 2009](#); [Hopkins, 2023](#); [Krishna et al., 2026](#)). This literature studies how agents use costly signals or investments to improve their rank or matching outcomes. In contrast, this paper studies how a designer allocates positional goods to induce agents' status.

2 Model

2.1 Setup

A monopolist seller (she) sells positional goods $x \in X \subseteq \mathbb{R}_{++}$ to a continuum of buyers (he) with unit mass. A larger x represents a higher-level good, such as a higher-end luxury good or a higher-priority boarding group, and the set X is determined by the seller.⁹ Let $x = 0 \notin X$ denote the outside option of not buying. Because consumers care about their relative position, consumption of positional goods confers *status* (defined later). Buyers are heterogeneous in their valuations of status (i.e., their types), denoted by θ , which has a

I refine some of their results by providing necessary and sufficient conditions and establish additional results in Appendix A.

⁹In general, it is without loss of generality to take $X = \mathbb{R}_{++}$. If the number of levels is constrained, then X is a finite subset of \mathbb{R}_{++} .

distribution $F(\theta)$ with continuous density $f(\theta)$ and full support on $\Theta = [0, \bar{\theta}]$.¹⁰ If a buyer with type θ buys the good at the price p , he obtains a payoff of $u(p, s, \theta) = \theta s - p + v(\theta)$, where $s \in [0, 1]$ is the status conferred by the good and $v(\theta) \geq 0$ is the intrinsic value of the good. Assume $v'(\theta) \geq 0$ and $v''(\theta) \leq 0$, so that higher types derive greater intrinsic value from the good at a diminishing marginal rate. If the buyer does not buy the good (i.e., $x = 0$), his status is defined as $s = 0$, and he does not obtain the intrinsic value, so his payoff is zero.

Equivalently, $-v(\theta)$ can be viewed as a type-dependent outside option, and the assumption implies that a higher type suffers more from not buying the good.

Definition of status. Let G denote the distribution of buyers' consumption over $x \in \tilde{X} \equiv X \cup \{0\}$, so $G(x)$ represents the mass of buyers who consume weakly lower-level goods than x or opt out. Following Robson (1992) and Hopkins and Kornienko (2004), I define the *status* of a buyer who consume $x \in X$ (i.e., participant) as

$$S(x, G(\cdot)) = \begin{cases} \frac{G^-(x) + G(x)}{2} \in [0, 1], & \text{if } x > 0 \\ 0, & \text{if } x = 0 \end{cases} \quad (1)$$

where $G^-(x) \equiv \lim_{x \rightarrow x^-} G(x)$ is the mass of buyers who consume *strictly* lower-level goods than x (or opt out). In other words, participants' status equals the mass of buyers who consume strictly lower-level goods plus half the mass of buyers at the same level.^{11,12} Non-participants ($x = 0$) obtains zero status and contribute to the status of every participant because they are ranked strictly below participants.

Because buyers only care about the induced status, the seller essentially sells status $s \in [0, 1]$, with distribution Ψ , subject to the feasibility constraint

$$s = \begin{cases} \frac{\Psi^-(s) + \Psi(s)}{2} \in [0, 1], & \text{if } s \in \text{supp}(\Psi) \setminus \{0\} \\ 0, & \text{if } s = 0. \end{cases}$$

For a given number of participants, their expected total status is constant, regardless of

¹⁰I allow $\bar{\theta} = \infty$ under the assumption that $\mathbb{E}[\theta] < \infty$. The lower bound can also be generalized to $\underline{\theta} > 0$, which leads to a complication in consumer surplus maximization under DFR (see Remark 4).

¹¹Hopkins and Kornienko (2004) assume a more general specification: $S(x, G(\cdot)) = \gamma G(x) + (1-\gamma)G^-(x) + \alpha$, where $\gamma \in [0, 1]$ captures the concern about the number of consumers at the same level, and $\alpha \geq 0$ is a constant representing a guaranteed minimum status for participants. My model incorporates α through the intrinsic value $v(\theta)$, and many results are robust to general γ (see Section 5.1.3).

¹²Without exclusion, this specification is equivalent to the one used in MSS and Dubey and Geanakoplos (2010) (up to an affine transformation): $S(x, G(\cdot)) = G^-(x) - (1 - G(x)) \in [-1, 1]$.

the distribution of status (whether it has mass points due to pooling). This reflects the zero-sum nature of status among participants.

This specification of status can arise from interpersonal comparisons (Frank, 1985b; Robson, 1992; Hopkins and Kornienko, 2004), especially for status goods. Consumers gain utility from outranking those who buy lower-level goods or cannot afford the good, but gain less from comparisons with consumers who purchase the same level.

In addition to psychological reasons, the specification can also arise from physical constraints. The following example illustrates this through priority services in queuing, where status corresponds to the reduction in waiting time.

Example 1 (Priority Services (Gershkov and Winter, 2023)). Consider a unit mass of consumers who arrive simultaneously, and a seller serves one consumer per unit time. The seller offers tiered priority services: consumers at higher priority levels are served first, and those at the same level are served in random order. A consumer's payoff is $u(p, t, \theta) = \tilde{v}(\theta) - \theta t - p$, where $t \in [0, 1]$ is the (expected) waiting time, θ is the cost of waiting, and $\tilde{v}(\theta)$ is the value of the good. Defining $s = 1 - t$ as the status (priority value) and $v(\theta) = \tilde{v}(\theta) - \theta$ as the net value of the good, this model is equivalent to the baseline model.¹³ In particular, excluded consumers contribute to participants' status because participants need not wait for those excluded.

More generally, education can also be viewed as a positional good where externalities arise from capacity constraints: an agent's education level determines his position in the job line (or social hierarchy), and his payoff depends on the number of individuals ahead of him or sharing the same position. This perspective connects naturally to the example of status contests in organizations.

Example 2 (Status Contests (Moldovanu, Sela, and Shi, 2007)). Consider a principal who maximizes agents' effort by designing status categories, where agents care about their relative position within the organization. The positional good x represents the status category, the price $p \geq 0$ represents the agent's effort, and the type θ represents the agent's ability (which determines the marginal cost of effort). Each agent has a linear effort cost p/θ and receives a payoff of $s - p/\theta$ upon attaining status $s \in [0, 1]$. An agent who is excluded receives zero payoff. After scaling agents' payoffs by θ , this is a special case of the model with $v(\theta) = 0$ and an additional constraint $p \geq 0$.

¹³The assumption $v'(\theta) \geq 0$ implies $\tilde{v}'(\theta) \geq 1$: higher types have higher net value of the good after accounting for waiting costs when served last. Section 5.4 considers an extension in which $v'(\theta) \leq -1$ (equivalently, $\tilde{v}'(\theta) \leq 0$), so that higher types derive lower net value from the good.

2.2 A Mechanism Design Approach

Consider a direct mechanism $(\chi(\theta, \omega), p(\theta), \sigma(\theta))$, consisting of a (potentially stochastic) allocation rule $\chi: \Theta \times \Omega \rightarrow \mathbb{R}_+$, a payment function $p: \Theta \rightarrow \mathbb{R}$, and a purchase indicator $\sigma: \Theta \rightarrow \{0, 1\}$. To allow for stochastic allocations, I introduce a random variable $\omega \in \Omega = [0, 1]$ to capture all randomness in the mechanism. The random variable ω is drawn uniformly, independent of types, and is common to all buyers.

In the direct mechanism, the buyer reports his type θ . If $\sigma(\theta) = 1$, the buyer pays $p(\theta)$ and receives the good $\chi(\theta, \omega) > 0$. If $\sigma(\theta) = 0$, the buyer does not purchase and receives the outside option 0. As a normalization, let $p(\theta) = 0$ and $\chi(\theta, \omega) = 0$ whenever $\sigma(\theta) = 0$, so χ incorporates the purchase decision.¹⁴ By the taxation principle, the direct mechanism can be implemented by a pricing scheme as a function of lotteries over the set of positional goods $X = \text{range}(\chi) \setminus \{0\}$.

Given an allocation χ , each realization $\omega \in \Omega$ induces a distribution over consumption levels $x \in X \cup \{0\}$, given by $G_\chi(x | \omega) = \int_\Theta \mathbf{1}_{\chi(\theta, \omega) \leq x} dF(\theta)$. Since buyers' payoffs depend on the allocation χ only through the status it confers, the allocation χ can be summarized by its induced status. Thus, it is equivalent to consider a direct mechanism $(s(\theta), p(\theta), \sigma(\theta))$, where $s: \Theta \rightarrow [0, 1]$ is the *status profile* induced by χ , defined as

$$s(\theta) = \mathbf{E}_\omega[S(\chi(\theta, \omega), G_\chi(\cdot | \omega))].$$

In particular, if $\sigma(\theta) = 0$, then $\chi(\theta, \omega) = 0$ for all $\omega \in \Omega$, and thus $s(\theta) = S(0, G_\chi) = 0$.

Let $U(\hat{\theta}|\theta) = \theta s(\hat{\theta}) - p(\hat{\theta}) + v(\theta)\sigma(\hat{\theta})$ denote the buyer's payoff when he reports $\hat{\theta}$ while his true type is θ . Define $U(\theta) = U(\theta|\theta)$. By the standard envelope argument, incentive compatibility requires that $U(\theta) = \max_{\hat{\theta}} U(\hat{\theta}|\theta)$, which implies that U is absolutely continuous with $U'(\theta) = s(\theta) + v'(\theta)\sigma(\theta)$ and the following lemmas.

Lemma 1. *A direct mechanism $(s(\theta), p(\theta), \sigma(\theta))$ is incentive-compatible if and only if*

- *there exists $\theta_0 \in [0, \bar{\theta}]$ such that $\sigma(\theta) = 1$ and $U(\theta) > 0$ for all $\theta > \theta_0$, and $\sigma(\theta) = 0$ and $U(\theta) = 0$ for all $\theta < \theta_0$;*
- *$s(\theta)$ is increasing;*
- *$U(\theta) = U(0) + \int_{\theta_0}^{\theta} (s(t) + v'(t)) dt$ for all $\theta \in [\theta_0, \bar{\theta}]$.*

¹⁴For any $\tilde{\chi}$ and \tilde{p} , one can always define $\chi = \tilde{\chi} \cdot \sigma$ and $p = \tilde{p} \cdot \sigma$ to incorporate the purchase decision. The purchase indicator σ is taken to be deterministic, which captures the assumption that the buyer obtains the intrinsic value $v(\theta)$ if and only if he purchases. When $v(\theta) = 0$, it is without loss to set $\sigma \equiv 1$ and allow stochastic nonparticipation by incorporating it to the allocation rule as $\chi(\theta, \omega) = 0$ and $p(\theta) = 0$.

The envelope condition implies that $U(\theta_0) = 0$ if $\theta_0 > 0$.

An incentive-compatible status profile $s(\theta)$ is not necessarily feasible, as it may be unable to be induced by an allocation χ . For example, a binary status $s(\theta) = \mathbf{1}_{\theta \geq \theta_0}$ cannot be induced by any allocation χ . Formally, say $s(\theta)$ is *feasible* if there exists an allocation $\chi: \Theta \times \Omega \rightarrow \mathbb{R}_+$ that induces it—i.e., such that $s(\theta) = \mathbf{E}_\omega[S(\chi(\theta, \omega), G_\chi(\cdot | \omega))]$. The following theorem provides the necessary and sufficient condition for an incentive-compatible (and hence increasing) status profile to be feasible.

Theorem 1 (Feasibility). *An increasing $s(\theta)$ is feasible if and only if there exists $\theta_0 \in [0, \bar{\theta}]$ such that $s(\theta)$ is majorized by $F(\theta)$ in quantile space on $[\theta_0, \bar{\theta}]$, denoted by $s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, that is,*

$$\int_{\theta}^{\bar{\theta}} s(\tilde{\theta}) dF(\tilde{\theta}) \leq \int_{\theta}^{\bar{\theta}} F(\tilde{\theta}) dF(\tilde{\theta}), \quad \text{for all } \theta \in [\theta_0, \bar{\theta}], \quad (2)$$

with equality at $\theta = \theta_0$, and $s(\theta) = 0$ for all $\theta \in [0, \theta_0)$.

Let $\text{MPS}_0(F)$ denote the set of increasing feasible status profiles. The proof (in Appendix B) is à la Theorem 3 (Border's Theorem) in KMS.

Remark 1. For deterministic allocations χ , a feasible $s(\theta)$ must be an extreme point of $\text{MPS}_0(F)$ —i.e., either the majorization constraint or the monotonicity constraint binds on $[\theta_0, \bar{\theta}]$ —and $s(\theta) = 0$ for all $[0, \theta_0)$. For stochastic allocations χ , a feasible $s(\theta)$ can be non-extreme points.¹⁵

Remark 2. When exclusion is impossible, we have $s \in \text{MPS}(F)$ and $\mathbf{E}[s] = \mathbf{E}[F] = 1/2$.¹⁶

To provide some intuition, first consider the full separation case where each participating buyer is assigned to a distinct tier. Each participant's status is just their quantile: $s(\theta) = F(\theta)$. Then, when buyers $\theta \in [\underline{\theta}_i, \bar{\theta}_i]$ are pooled into the same tier $x_i > 0$, the status of that tier becomes the average quantile of the buyers in the pool because

$$s(\theta) = \frac{G^-(x_i) + G(x_i)}{2} = \frac{F(\underline{\theta}_i) + F(\bar{\theta}_i)}{2}, \quad \text{for all } \theta \in [\underline{\theta}_i, \bar{\theta}_i].$$

Thus, pooling coarsens the status profile across buyers, while preserving the aggregate status, making their status a mean-preserving spread of the quantile. Randomization

¹⁵If the seller can randomly not serve participants, the feasibility condition becomes: s is weakly majorized by F in quantile space, denoted by $s \in \text{MPS}_w(F)$. However, the model assumes that types above (below) the cutoff θ_0 always (never) purchase and obtain an intrinsic value $v(\theta)$. If $v(\theta) = 0$, this assumption has no bite (see also Footnote 14).

¹⁶Under MSS's specification $S(x, G) = G^-(x) - (1 - G(x))$ without exclusion, the feasibility condition becomes $s \in \text{MPS}(2F - 1)$ with $\mathbf{E}[s] = \mathbf{E}[2F - 1] = 0$.

also preserves this relationship for each realization ω and expands the set of feasible status profiles to the exact set of mean-preserving spreads through convexification.

Example 3 (Two tiers). Suppose there are two levels of positional goods, (p_L, s_L) and (p_H, s_H) . Assume $p_L = 0$ so that every buyer participates ($\theta_0 = 0$). Let θ^* denote the type who is indifferent between the two levels, which is determined by p_H . Then, the induced status profile is

$$s(\theta) = \begin{cases} s_L = F(\theta^*)/2, & \text{if } \theta < \theta^*, \\ s_H = (1 + F(\theta^*))/2, & \text{otherwise.} \end{cases}$$

It is straightforward to check that $\mathbf{E}[s] = 1/2$ and that $\int_{\theta}^{\bar{\theta}} s(t) dF(t) \leq \int_{\theta}^{\bar{\theta}} F(t) dF(t)$ for all $\theta \in [0, \bar{\theta}]$, with equality at $\theta = 0, \theta^*$, and $\bar{\theta}$.

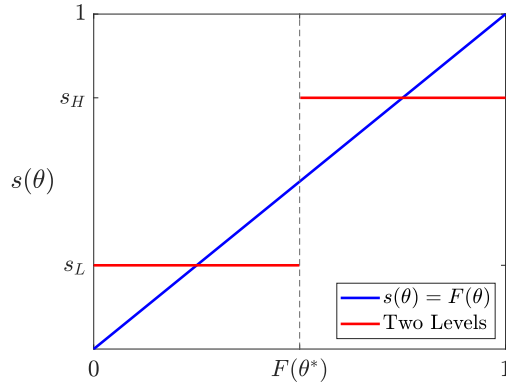


Figure 1: Two Priority Levels

It is worth noting that a binary status profile $s(\theta) = \mathbf{1}_{\theta \geq \theta^*}$, which resembles a posted-price mechanism when selling ordinary goods, is infeasible because it violates the mean-preserving spread condition at the top. Intuitively, this is because a good meant to confer the highest status $s = 1$ ceases to do so once sold to multiple buyers.

Finally, when the seller is constrained to offer at most $n \geq 1$ levels of positional goods, so that $|X| \leq n$, the feasibility condition is more restrictive. Say that a status profile $s(\theta)$ is *n-feasible* if there exists an allocation $\chi: \Theta \times \Omega \rightarrow X \cup \{0\}$, with $|X| \leq n$, that induces it. If χ is deterministic, then any *n-feasible* incentive-compatible (and hence increasing) $s(\theta)$ is a step function on $[\theta_0, \bar{\theta}]$, generated by partitioning participating types into at most n pooling intervals. If stochastic allocations are allowed, then the set of increasing *n-feasible* $s(\theta)$ is the convex hull of the set of such step functions.

3 Revenue Maximization

In this section, I characterize the optimal mechanism that maximizes the seller's revenue and study its welfare implications. Two extreme allocations are particularly of interest: full separation of participants (i.e., $s(\theta) = F(\theta) \cdot \mathbf{1}_{\theta \geq \theta_0}$ at different prices) and pooling among participants (i.e., $s(\theta) = \frac{1+F(\theta_0)}{2} \cdot \mathbf{1}_{\theta \geq \theta_0}$ at the same price). The former can be induced by offering a continuum of positional goods, and the latter can be induced by offering a single good (tier).

3.1 Seller's Problem

The revenue maximization problem is given by

$$\max_{s(\theta), p(\theta), \theta_0} \int_{\theta_0}^{\bar{\theta}} p(\theta) dF(\theta) \quad (3)$$

subject to the following constraints for all $\theta \in [\theta_0, \bar{\theta}]$:

$$U(\theta) \equiv \theta s(\theta) - p(\theta) + v(\theta) \geq 0 \quad (\text{IR}) \quad (4)$$

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} (s(t) + v'(t)) dt \quad (\text{IC}) \quad (5)$$

$$s(\theta) \text{ is increasing} \quad (6)$$

$$s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]}) \quad (\text{MPS}) \quad (7)$$

Define $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$ and $J_v(\theta) = v(\theta) - \frac{1-F(\theta)}{f(\theta)}v'(\theta)$. Say F satisfies Myerson's regularity if $J(\theta)$ is increasing. By standard arguments, expected revenue is given by

$$R = \int_{\theta_0}^{\bar{\theta}} (J(\theta)s(\theta) - U(\theta_0)) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) \quad (8)$$

It is optimal to set $U(\theta_0) = 0$ by setting $p(\theta_0) = \theta_0 s(\theta_0) + v(\theta_0) \geq 0$. Thus, the revenue maximization problem is equivalent to

$$\max_{s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})} \int_{\theta_0}^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) = \int_{\theta_0}^{\bar{\theta}} (J(\theta)s(\theta) + J_v(\theta)) dF(\theta) \quad (9)$$

For any $\theta_0 \in [0, \bar{\theta}]$, the objective is a continuous linear functional of s over $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$. By KMS, this set is convex and compact, so Bauer's maximum principle implies that the maximum is attained at an extreme point, which is induced by a deterministic allocation.

For any two increasing status profiles $s, \hat{s} \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, say s is *finer* than \hat{s} (in terms of majorization) if $\hat{s} \in \text{MPS}(s)$ —i.e., $s \succ \hat{s}$ in the majorization order.¹⁷ If $J(\theta)$ is increasing, by the Fan-Lorentz Theorem (see Theorem 4 in KMS) implies that the seller can increase revenue by offering a mechanism with a finer status profile. Hence, we have the following proposition.

Proposition 2. *If $J(\theta)$ is increasing on Θ , then*

(i) *The seller’s revenue increases as she offers more levels of positional goods (and sets the price and size for each tier optimally).¹⁸*

(ii) *The revenue-maximizing mechanism excludes $\theta < \theta_0^*$ and fully separates $\theta \geq \theta_0^*$:*

$$s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*], \quad p^*(\theta) = \left(\theta_0^* F(\theta_0^*) + v(\theta_0^*) + \int_{\theta_0^*}^{\theta} t \, dF(t) \right) \cdot \mathbf{1}[\theta \geq \theta_0^*],$$

where the optimal cutoff type is $\theta_0^* = \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) F(\theta) \, dF(\theta) + v(\theta_0)(1 - F(\theta_0))$.

If $J(\theta)$ is not monotonic, define $\tilde{J}(\theta; \theta_0) = \int_{\theta_0}^{\theta} J(t) \, dF(t)$ and let $K(\tau) = \text{conv } \tilde{J}(F^{-1}(\tau))$ denote its convex hull on $[F(\theta_0), 1]$ in quantile space. Then, the revenue-maximizing mechanism excludes $\theta < \theta_0^*$, pools types into the same positional good level if K is affine, and separates types if $K \circ F = \tilde{J}$, where $\theta_0^* = \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} F(\theta) \, dK(F(\theta)) + v(\theta_0)(1 - F(\theta_0))$.

If the type distribution satisfies Myerson’s regularity condition, the revenue-maximizing mechanism can be implemented by an all-pay auction with a reserve price, where buyers who pay more, conditional on meeting the reserve price, receive higher levels of positional goods. In applications such as education or status within organizations, this may be interpreted as an all-pay contest with a minimum effort requirement.

In general, the optimal mechanism is derived by applying the ironing technique to obtain the convex hull of $\tilde{J}(\theta)$ in quantile space (Myerson, 1981; Toikka, 2011). If the virtual valuation $J(\theta)$ is decreasing for some types, the revenue-maximizing mechanism pools them into the same positional good level along with some adjacent types.

The results have several implications. First, under Myerson’s regularity condition, it is optimal to offer as many levels of positional goods as possible. As the number of levels increases, the seller can induce a finer status profile, which not only generates efficiency gains but also allows the seller to extract higher revenue. For example, airlines

¹⁷If s induces a finer partition of types than \hat{s} , then s is *finer* than \hat{s} (in terms of majorization); the converse is not true.

¹⁸Formally, let R_n denote the seller’s maximal revenue when she offers exactly n levels. Then, if $J(\theta)$ is (strictly) increasing, R_n is (strictly) increasing in n .

should introduce more boarding groups or flight classes, and luxury companies should proliferate products from factory stores to high-end exclusives. Of course, offering more levels of positional goods may involve higher costs, as discussed in the next subsection.

Compared to selling ordinary goods with intrinsic value $v(\theta)$ only, attaching status value benefits the seller, which may explain why luxury companies invest heavily in marketing to cultivate status value.

In addition, the seller also benefits from excluding low types. The marginal revenue of including more types is given by $(J(\theta)F(\theta) + J_v(\theta))f(\theta)$. If this marginal revenue single-crosses zero from below, the revenue is quasi-concave in θ_0 , so the optimal mechanism excludes all types with negative marginal revenue. In particular, when the intrinsic value $v(\theta) = 0$, all types with negative virtual valuation are excluded. The following corollary characterizes properties of the optimal exclusion.

Corollary 2.1. *If $J(\theta)$ is increasing, the optimal cutoff θ_0^* has the following properties:*

(i) *Intrinsic value reduces exclusion: $\theta_0^* \leq \hat{\theta}_0 := J^{-1}(0)$.¹⁹ The equality holds if $v(\theta)$ is linear (i.e., $v(\theta) = \alpha\theta$ for some $\alpha \geq 0$).*

(ii) *If $v(0) \geq (v'(0) + 1)/f(0)$, then no exclusion is optimal—i.e., $\theta_0^* = 0$.²⁰*

Under the regularity condition, the presence of intrinsic value $v(\theta)$ reduces the optimal exclusion, particularly when it is nonlinear. From another perspective, the optimal exclusion is lower than that in selling ordinary goods with intrinsic value θq and without status value (Mussa and Rosen, 1978), where agents with negative virtual valuation $J(\theta) < 0$ are excluded. In addition, when the intrinsic value $v(0)$ is large enough, and if there are many low types ($f(0)$ is large), it is optimal for the seller to serve every type.

3.2 Approximation

In practice, the number of levels may be constrained by regulation, implementation costs, etc. The following proposition shows that a posted-price mechanism with a single tier of positional goods (and an optimally chosen price) obtains at least half of the unrestricted maximal revenue.

Proposition 3. *The seller can obtain at least half the maximum revenue by selling a single tier.*

¹⁹If there are multiple values of θ for which $J(\theta) = 0$, define $\hat{\theta}_0 = \inf\{\theta \in [0, \bar{\theta}] : J(\theta) > 0\}$.

²⁰Another sufficient condition is that $J(\theta)F(\theta) + J_v(\theta)$ is increasing and $v(0) \geq v'(0)/f(0)$.

Proof sketch. For ease of exposition, assume $v(\theta) = 0$, so that the buyer's payoff is $u(p, s) = \theta s - p$. Consider an auxiliary problem of selling an indivisible ordinary good, in which $u(p, q) = \theta q - p$ when the buyer receives the good with probability $q \in [0, 1]$. In the auxiliary problem, the set of incentive-compatible allocations is $\mathcal{M} = \{q: \Theta \rightarrow [0, 1] \mid q \text{ is increasing}\}$, and the optimal mechanism is a posted price $p^* = \arg \max_p p(1 - F(p))$ (i.e., $q^*(\theta) = \mathbf{1}[\theta \geq p^*]$). Let $R_{aux}^* = p^*(1 - F(p^*))$ denote the maximum revenue.

Let R^* denote the maximum revenue from selling positional goods. When selling positional goods, because the set of feasible status profiles is a subset of \mathcal{M} , we have $R^* < R_{aux}^*$.²¹ Let R_1 denote the maximum revenue from selling a single tier of positional good. Selling a single tier at price $\frac{1+F(p^*)}{2}p^*$ excludes buyers with types $\theta < p^*$, as in the auxiliary problem, and guarantees each participant at least $s = \frac{1+F(p^*)}{2} \geq \frac{1}{2}$. The resulting revenue is thus at least $\frac{1}{2}R_{aux}^*$. Hence, we have $R_1 \geq \frac{1}{2}R_{aux}^* > \frac{1}{2}R^*$.

The complete proof, which allows $v(\theta) \geq 0$, is given in Appendix B. \square

Remark 3. The lower bound does not require IFR or Myerson's regularity.

The auxiliary problem of selling indivisible ordinary goods highlights the distinction between selling ordinary goods and positional goods. In the auxiliary problem, selling an item to one consumer has no externalities on others, so the seller can allocate $q = 1$ to multiple buyers. By contrast, when selling positional goods, because consumers care about their relative positions, negative externalities arise—the seller cannot allocate the highest status $s = 1$ to multiple buyers. Thus, the seller of positional goods is subject to an additional feasibility constraint, resulting in a strictly lower revenue.

When there are costs associated with offering multiple levels of positional goods, a finite number of levels is optimal. Formally, when the seller offers $n \geq 1$ levels of positional goods, let R_n denote the maximal revenue and $C_n > 0$ denote the cost of offering n levels. If C_n is strictly increasing and (weakly) convex in n , the optimal number of levels is finite, since R_n is bounded above by R^* . In addition, if $C_2 - C_1 \geq R^*/2$, then Proposition 3 implies that a single tier is optimal.

Graphical Illustration. Figure 2 plots the *revenue curve* $R(\tau) = (1 - \tau)F^{-1}(\tau)$ in quantile space $\tau = F(\theta)$. For exposition, I focus on the regular case where $J(\theta)$ is increasing, so that the revenue curve is concave, and assume the intrinsic value $v(\theta) = 0$.

The *blue* area in Figure 2 represents R^* , the revenue generated by the revenue-maximizing allocation $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$. To see this, note that $R'(\tau) = -J(\tau)$,

²¹The inequality is strict because any optimal mechanism in the auxiliary problem assign $q = 1$ to the highest types, which is infeasible when selling positional goods.

so substituting $\tau = F(\theta)$ and integrating by parts, the maximum revenue is

$$\int_{\theta_0^*}^{\bar{\theta}} J(\theta)F(\theta) dF(\theta) = - \int_{\tau_0^*}^1 \tau dR(\tau) = \tau_0^*R(\tau_0^*) + \int_{\tau_0^*}^1 R(\tau) d\tau,$$

which equals the area of the blue region.

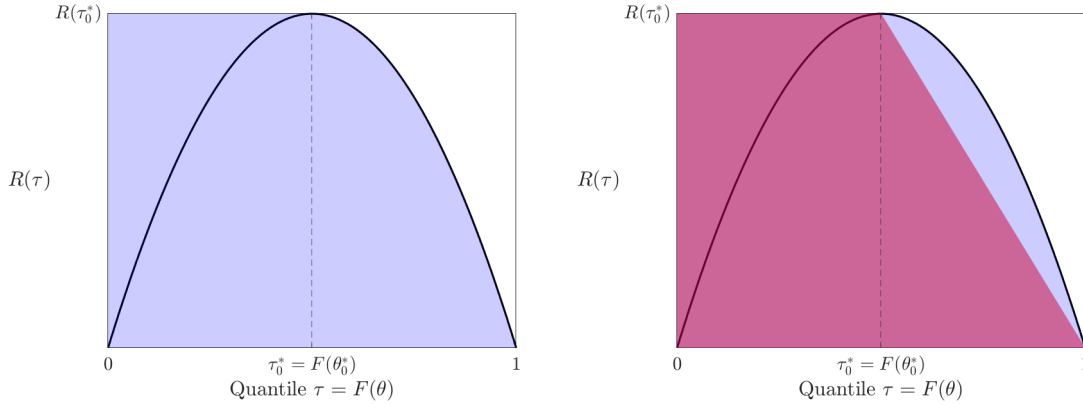


Figure 2: Revenue curve $R(\tau)$ for the uniform distribution

The *red* trapezoid in the right panel represents the revenue from selling a single good at price $p_0^* = \frac{1+F(\theta_0^*)}{2}\theta_0^*$, which induces $\bar{s}(\theta) = \frac{1+F(\theta_0^*)}{2} \cdot \mathbf{1}[\theta \geq \theta_0^*]$. This is because the revenue under \bar{s} is

$$\int_{\theta_0^*}^{\bar{\theta}} J(\theta) \frac{1+F(\theta_0^*)}{2} dF(\theta) = \frac{1+\tau_0^*}{2} R(\tau_0^*),$$

which equals the area of the red trapezoid in the right panel. Since the seller can set the optimal price and p_0^* is a feasible option, the revenue from selling a single tier is at least the red area.

Finally, the entire box, which has area $R(\tau_0^*)$, represents the maximum revenue from selling ordinary goods in the auxiliary problem. It is straightforward to verify that the blue area is strictly smaller than the entire box, and that the red area is at least half of the entire box. Hence, the red area is strictly larger than half of the blue area.

The approximation is close for many common distributions, as shown in the following examples with $v(\theta) = 0$.

Example (Exponential distribution). If $F(\theta) = 1 - \exp(-\lambda\theta)$ where $\lambda > 0$, selling a single good can obtain 91.9% of the maximum revenue.

Example (Uniform distribution). If $F(\theta) = \theta$ on $[0, 1]$, selling a single good can obtain 92.4% of the maximum revenue.

Example (Power distribution). Assume $F(\theta) = \theta^\beta$ on $[0, 1]$. If $\beta \geq 1$, $J(\theta)$ is strictly increasing, so the optimal mechanism separates participants.

If $\beta < 1$, although $J(\theta)$ is single-dipped, because the $J(\theta)$ single-crosses zero from below and is increasing whenever $J(\theta) > 0$, the optimal mechanism still separates participants. In both cases, the approximation ratio is

$$\frac{R_1}{R^*} = \frac{(1 + \beta) \left(\frac{1}{1+2\beta} \right)^{\frac{1}{2\beta}}}{\beta + (1 + \beta)^{-1 - \frac{1}{\beta}}} > 91.4\%.$$

The ratio approaches 1 as $\beta \rightarrow \infty$ (i.e., F is sufficiently convex) or $\beta \rightarrow 0$ (i.e., F is sufficiently concave).

Example (Pareto distribution). If $F(\theta) = 1 - \theta^{-\beta}$ on $[1, \infty)$ where $\beta > 1$ (so that the mean is finite), the approximation ratio is

$$\frac{R_1}{R^*} = \left(1 - \frac{1}{2\beta - 1} \right)^{\frac{\beta-1}{\beta}} > 79.29\%.$$

3.3 Welfare Comparative Statics

Positional goods are often subject to regulations or exogenous constraints on the number of levels and service coverage, which may be motivated by consumer welfare concerns. I now analyze how consumer welfare responds to these exogenous constraints, given that the seller maximizes revenue subject to these constraints.

The directions of these welfare effects are *a priori* ambiguous. When more levels of positional goods are available, under the regularity condition, the seller optimally offers a finer status profile. A finer status profile generates efficiency gains by assigning higher status to consumers who value it more, which may raise consumer welfare. However, as the seller charges higher prices for higher levels, she may extract more surplus, potentially reducing consumer welfare.

Expanding coverage (by reducing the price of the lowest tier) also creates a tradeoff. Assume that the seller offers a single tier, and consider the effect of lowering its price. On the extensive margin, expanding coverage always benefits new participants who previously could not afford it. On the intensive margin, although existing consumers pay a lower price, expanding coverage also reduces their status because they need to be pooled with new consumers. For example, when more buyers gain access to a luxury good, the status conferred by the entry-level good may decline. When a service provider serves more customers, those at the basic paid tier may face longer wait times.

Moreover, these two effects are interdependent. When the seller offers more tiers, she may attract additional consumers, thereby expanding coverage. Conversely, when she expands coverage by lowering the price of the lowest tier, both consumers' choices and the seller's pricing among the higher tiers may vary, which in turn changes the status profile. To isolate the welfare effects of each policy dimension, I conduct comparative statics by varying one dimension at a time, while holding the other fixed.

Using the envelope theorem, consumer surplus can be written as

$$W = \int_{\theta_0}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\theta_0}^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} (s(\theta) + v'(\theta)) + U(\theta_0) \right) dF(\theta). \quad (10)$$

As the cap on the number of tiers changes, the seller optimizes the mechanism in response to the change, and it is optimal to adjust the pricing scheme so that $U(\theta_0) = 0$. Thus, holding exclusion fixed, the effect on consumer surplus is determined by the failure rate $\frac{f(\theta)}{1-F(\theta)}$. Say F satisfies IFR (DFR) if it has an increasing (decreasing) failure rate.

For expanding coverage, I consider the effect of reducing $\theta_0 > 0$ by lowering the price of the lowest tier in two cases: when the number of tiers is unrestricted or when it is fixed at one. These two cases naturally fix the status of higher-tier consumers and simplify the analysis. Since $\theta_0 > 0$, Lemma 1 implies $U(\theta_0) = 0$, and the effect may also depend on the failure rate.

Proposition 4. *Assume $J(\theta)$ is increasing. The following comparative statics hold.*

(i) *Holding exclusion fixed, as more levels are available to the seller, consumer surplus decreases (increases) if F satisfies IFR (DFR).²²*

(ii) *When the number of tiers is unrestricted, expanding coverage by lowering the price of the lowest tier increases consumer surplus.*

When the seller is restricted to a single tier, expanding coverage increases consumer surplus if F satisfies IFR (and can decrease surplus otherwise).

The proposition highlights the tension between revenue and consumer surplus under IFR: holding participation fixed, as the seller offers more levels of positional goods (and sets the price and size of each level optimally), revenue increases while consumer surplus decreases. Although adding levels leads to a finer status profile and creates efficiency gains, the seller extracts more surplus than she generates, leaving consumers worse off. Therefore, when exclusion is fixed, limiting the number of levels available to the seller benefits consumers under IFR.

²²In particular, when the lowest tier is free, there is no exclusion (see Appendix A.2)

The opposite is true if the distribution has a *decreasing* failure rate. A finer status profile shifts status from lower types to higher types. While higher types value status more, this shift does not automatically benefit them because the seller also extracts more surplus from them through higher prices. DFR matters in two ways. First, it makes types in the upper tail sufficiently dispersed, so improving the status of very high types creates information rents for them rather than being fully dissipated through higher prices. Second, DFR implies that the inverse hazard rate is increasing, so aggregate consumer surplus places greater weight on the gains of higher types than the losses of lower types. Hence, a finer status profile raises aggregate consumer welfare. Consequently, if the distribution also satisfies Myerson's regularity (e.g., Pareto with a finite mean), allowing the seller to offer more levels benefits both the seller and consumers.

In practice, exclusion is fixed (to zero), for instance, when there is a free basic tier available. If exclusion is *not* fixed, however, the comparative static becomes ambiguous because when the seller offers more levels, consumer participation may also increase, which affects consumer surplus. For example, when $F(\theta) = \theta$ and $v(\theta) = 0$, the seller excludes $\theta < 1/\sqrt{3} \approx 0.58$ when constrained to offer a single tier, and excludes $\theta < 0.5$ when unconstrained. Consequently, consumer surplus is higher in the latter case despite more levels being offered, as the gain from increased participation outweighs the loss from finer status. In general, the aggregate effect depends on the distribution of consumer types, and there is no simple condition for its sign. This result highlights the difficulty of regulating the number of positional-good levels: unless the regulator also controls the exclusion level (for example, by mandating a free basic service), it is unclear whether such regulation benefits consumers.

When the number of levels is unconstrained, the seller fully separates participants if F satisfies Myerson's regularity. Thus, expanding coverage benefits new participants without lowering the status of existing ones, so it benefits consumers unambiguously.

When the seller is constrained to offer a single tier, the welfare effect becomes more complicated. If F satisfies IFR, expanding coverage by lowering its price increases consumer surplus. Perhaps surprisingly, for general distributions, expanding coverage may *decrease* consumer surplus. The reason is that while expanding coverage benefits the additional participants, it also reduces the status of current participants by pooling them with the newcomers. When the type distribution has a thin tail, implied by IFR, the gain on the extensive margin outweighs the loss to current participants on the intensive margin. In general, however, the loss to current participants may dominate, leading to a decrease in consumer surplus.

Finally, when the seller is restricted to $n \in (1, \infty)$ tiers, the welfare effect of expanding

coverage is ambiguous even under IFR. The reason is that when the seller expands coverage by lowering the price of the lowest tier, the prices and cutoffs of the higher tiers can also vary, thereby changing the status of consumers at those tiers. The boundary cases $n = \infty$ and $n = 1$ shut down this other dimension by fixing the status of higher-tier consumers (or making it vacuous), and are therefore tractable.

4 Welfare Maximization

In this section, I study the optimal mechanism that maximizes welfare. Without positional externalities, the welfare-maximizing mechanism is to allocate the good to all consumers. For positional goods, however, the value of the good diminishes as more consumers receive it, so the welfare-maximizing mechanism may involve offering multiple levels of goods and excluding some consumers.

First, I consider consumer surplus maximization when negative transfers (i.e., subsidies) are allowed and subject to budget balance. Then, I study consumer surplus maximization under the constraint that transfers are nonnegative, which is relevant for applications where subsidies are unavailable or where the price represents effort. Finally, I characterize the optimal mechanism that maximizes social welfare—a weighted sum of revenue and consumer surplus.

4.1 Consumer Surplus Maximization

I first consider consumer surplus maximization when negative transfers are allowed and subject to budget balance (i.e., $\int_{\theta_0}^{\bar{\theta}} p(\theta) dF(\theta) = 0$). Then, substituting the budget balance constraint, consumer surplus becomes

$$W = \int_{\theta_0}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\theta_0}^{\bar{\theta}} (\theta s(\theta) + v(\theta)) dF(\theta). \quad (11)$$

It is straightforward that consumer surplus is maximized by full separation without exclusion (and with cross-subsidization). Intuitively, this is because assortative matching is efficient, and a fixed subsidy can redistribute efficiency gains among consumers while maintaining incentive compatibility.

Observation 1. When negative transfers are allowed and subject to budget balance, the consumer surplus-maximizing mechanism is $s(\theta) = F(\theta)$ and $p(\theta) = \int_0^\theta t dF(t) - \mathbf{E}[(1 - F(\theta))\theta]$.

Under this mechanism, higher types pay more to attain higher status than lower types. The payments will be used to cross-subsidize lower types, who accept lower status but are compensated by a subsidy. This mechanism can be implemented through an all-pay auction (without a reserve price) with a lump-sum subsidy of $\mathbf{E}[(1 - F(\theta))\theta]$, where buyers who pay more receive higher positions, and every buyer receives the same subsidy financed by the auction payments. For sufficiently low types, the transfer $p(\theta)$ is negative, resulting in a net subsidy.

4.2 Consumer Surplus Maximization without Subsidies

The result above relies on negative transfers, which may be infeasible in many applications. In particular, for luxury or status goods, it is implausible to use subsidies. Moreover, in applications to organizational hierarchies and education, the “price” represents effort and must be nonnegative. Imposing a nonnegative price constraint $p(\theta) \geq 0$ significantly changes the optimal mechanism.

To maximize consumer surplus under the nonnegative price constraint, we need to pin down $U(0)$ in equation (10):

$$W = \int_{\theta_0}^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} (s(\theta) + v'(\theta)) + U(\theta_0) \right) dF(\theta). \quad (12)$$

If the cutoff type $\theta_0 > 0$, then $U(\theta_0) = 0$. Otherwise, if $\theta_0 = 0$, because $U(0) = v(0) - p(0)$, consumer surplus under the nonnegative price constraint ($p(\theta) \geq 0$) is maximized by $p(0) = 0$ (so that $U(0) = v(0)$ and $p(\theta) = \int_0^\theta t ds(t) \geq 0$). Since $U(\theta_0)$ is pinned down, the optimal mechanism is again determined by the failure rate.

Proposition 5. *Under the nonnegative price constraint, consumer surplus is maximized by*

- (i) *total pooling (i.e., $s(\theta) = 1/2$) and $p(\theta) = 0$ if F satisfies IFR,²³*
- (ii) *full separation (i.e., $s(\theta) = F(\theta)$) and $p(\theta) = \int_0^\theta t dF(t)$ if F satisfies DFR;*
- (iii) *bottom pooling and top separation, with $p(\theta) = \int_0^\theta t ds(t)$, if the failure rate $\frac{f(\theta)}{1-F(\theta)}$ is single-peaked;*

²³The necessary and sufficient condition is:

$$\int_0^\theta \left(\frac{1 - F(t)}{f(t)} - \mathbf{E}[\theta] \right) dF(t) \geq 0 \quad \text{for all } \theta \in [0, \bar{\theta}].$$

which holds if $\frac{1-F(\theta)}{f(\theta)}$ single-crosses $\mathbf{E}[\theta]$ from above.

(iv) *bottom separation and top pooling, with $p(\theta) = \int_0^\theta t ds(t)$, if the failure rate $\frac{f(\theta)}{1-F(\theta)}$ is single-dipped.*

Remark 4. If the lowest type is changed to $\underline{\theta} > 0$, then some pooling at the bottom can increase consumer surplus even when the failure rate is decreasing, as pooling at the bottom increases $U(\underline{\theta}) = \underline{\theta}s(\underline{\theta}) + v(\underline{\theta}) > 0$ by raising the lowest status $s(\underline{\theta})$.

Under the nonnegative price constraint, the consumer-optimal mechanism does not exclude consumers. The result is in contrast to Proposition 4, where exclusion may increase consumer surplus under DFR, because Proposition 5 focuses on the consumer-optimal mechanism. Under IFR, because it is optimal to pool participants, exclusion raises the status of current participants, who would otherwise be pooled with more consumers. Nevertheless, the loss to excluded consumers on the extensive margin exceeds the gain to participants on the intensive margin, so the net effect is detrimental. Under DFR, exclusion has no benefit to current participants because the optimal mechanism does not pool them with additional participants.²⁴

Under IFR, the consumer-optimal regulation of positional goods without using subsidies is to restrict the seller to offer a single good and serve all consumers for free, such as a free regular service. The results reverse under DFR, for the same reason as Proposition 4. For nonmonotone failure rates, the consumer-optimal mechanism can have pooling and separating regions. When the failure rate is single-peaked, such as the log-normal distribution, the optimal mechanism pools low types and separates high types. When the failure rate is single-dipped, such as the power distribution $F(\theta) = \theta^\beta$ with $\beta \in (0, 1)$, the optimal mechanism separates low types and pools high types.

The results also have implications for organizational hierarchies and education. If the ability distribution satisfies IFR, which implies a thin tail, eliminating status competition entirely is optimal for agents' welfare. In the education application, this implies complete educational disarmament—for example, admission lotteries when allocating seats at sought-after schools. If instead there are a few superstars in the upper tail (e.g., under generalized Pareto distributions), meritocracy is optimal for agents' welfare.²⁵ Intuitively, while meritocracy creates a rat race that forces all agents to exert effort, superstars can benefit from it because they have lower marginal costs of effort and are sufficiently spread out in the upper tail. Although low-ability agents still suffer, the heavy tail also makes

²⁴Although exclusion also changes prices, the price adjustments are pinned down by incentive compatibility through revenue equivalence, so the welfare comparison need only consider status and the exclusion level.

²⁵A generalized Pareto distribution with shape parameter $\xi \in (0, 1)$ has a distribution function $F(\theta) = 1(1 + \xi\theta)^{-1/\xi}$ on the support $[0, \infty)$, which has finite mean and satisfies DFR.

the gain to higher-ability agents dominate in the aggregate. Hence, more (less) intense competition is optimal for agents' welfare when the ability distribution has a heavy (thin) tail.

The dependence on the failure rate is reminiscent of optimal bidding rings without transfers in McAfee and McMillan (1992). Indeed, both results stem from assumptions that pin down $U(\theta_0)$. An analog of their results also applies to this setting if agents can collude and transfers are impossible. In the application to organizational hierarchies or education, optimal collusion is collective disarmament under IFR, where all agents invest the minimal effort and share the same status, and status competition under DFR.

4.3 Social Welfare Maximization

In many settings, a social planner maximizes a weighted sum of the seller's revenue and consumer surplus. Let $W_S = \mathbf{E}[\lambda p(\theta) + U(\theta)]$ (where $\lambda \geq 0$) denote expected social welfare. In applications to monopolist provision of positional goods, p is monetary transfer, and λ is the welfare weight on the seller's revenue relative to consumer surplus. When $\lambda \geq 1$, the social planner puts higher weight on the seller's revenue than consumer surplus.

In applications to education or status within organizations, $p \geq 0$ is nonnegative effort, and λ captures the degree to which effort is productive from the social perspective. When $\lambda = 0$, the agents' effort is purely signaling. As λ increases, effort becomes more productive, and the social planner has greater incentives to induce agents to exert effort.

If negative transfers are allowed and subject to budget balance ($\mathbf{E}[p(\theta)] = 0$), then $W_S = \mathbf{E}[U(\theta)]$, so full separation without exclusion, as shown in Observation 1, also maximizes social welfare. Therefore, the more interesting case to consider is when budget balance is not required, or when negative transfers are not allowed. In the former case, I assume the welfare weight $\lambda \geq 1$, since otherwise a transfer from the seller to consumers would always increase social welfare without bound.

Define the weighted virtual value as $J_\lambda(\theta) = \lambda\theta - (\lambda - 1)\frac{1-F(\theta)}{f(\theta)}$. The expected social welfare is

$$W_S = \int_{\theta_0}^{\bar{\theta}} J_\lambda(\theta)s(\theta) dF(\theta) + \int_{\theta_0}^{\bar{\theta}} \left(\lambda v(\theta) - (\lambda - 1)v'(\theta)\frac{1-F(\theta)}{f(\theta)} - (\lambda - 1)U(\theta_0) \right) dF(\theta).$$

In particular, if $\lambda = 1$, transfers are welfare-neutral, and we have $J_\lambda(\theta) = \theta$. The social welfare is maximized by full separation without exclusion because a finer status profile strictly increases social welfare by matching the higher types with higher status.

If $\lambda > 1$, the social planner puts higher weight on the revenue, so it is optimal to set

$U(\theta_0) = 0$, and as in revenue maximization, the payment identity implies that the price is nonnegative. If $\lambda < 1$ and negative transfers are not allowed, analogous to the consumer surplus maximization without subsidies (Section 4.2), we can also pin down $U(0) = v(0)$ if $\theta_0 = 0$ and $U(\theta_0) = 0$ if $\theta_0 > 0$. Therefore, we have the following result.

Proposition 6. *Assume $J_\lambda(\theta)$ is increasing. If $\lambda \geq 1$ or if negative transfers are not allowed, the social welfare-maximizing mechanism fully separates the participants and excludes types below a certain threshold.*

In general, the social welfare-maximizing mechanism depends on the welfare weight $\lambda \geq 0$ on the seller relative to consumers, or the extent to which effort is productive relative to signaling.

5 Extensions

5.1 Alternative Specifications of Status

5.1.1 Convex or Concave Status

When positional externalities arise from interpersonal comparison, a concave or convex specification of status may arise. Assume $\phi: [0, 1] \rightarrow [0, 1]$ is strictly increasing, continuously differentiable, and satisfies $\phi(0) = 0$, and define the status function as

$$\tilde{S}(x, G) = \phi(S(x, G)) = \phi\left(\frac{G^-(x) + G(x)}{2}\right). \quad (13)$$

Recall that the original feasible set is $\text{MPS}_0(F) = \{s: \Theta \rightarrow [0, 1] \text{ increasing} \mid \exists \theta_0 \in [0, \bar{\theta}] \text{ such that } s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]}) \text{, and } s(\theta) = 0 \text{ for all } \theta \in [0, \theta_0]\}$. Define

$$\mathcal{F} = \{\tilde{s}: \Theta \rightarrow [0, 1] \mid \tilde{s} = \phi \circ s, s \in \text{ext MPS}_0(F)\}$$

as the set of status profiles obtained by passing the extreme points of the original feasibility set through ϕ . Let $\text{conv } \mathcal{F}$ denote the convex hull of \mathcal{F} .²⁶ Under this specification, an increasing status profile \tilde{s} is feasible if and only if $\tilde{s} \in \text{conv } \mathcal{F}$.

The following proposition shows that the main results are robust to nonlinear status, although stronger regularity conditions may be required. The proof uses two key observations. First, because the objective is linear in \tilde{s} , it is without loss of optimality to restrict

²⁶Equivalently, $\text{conv } \mathcal{F}$ is the set of functions that can be expressed as a convex combination of functions in \mathcal{F} , that is, $\tilde{s} = \sum_{i=1}^n \lambda_i \phi \circ s_i$ for some $s_i \in \text{ext MPS}_0(F)$ and $\lambda_i \geq 0$ with $\sum_{i=1}^n \lambda_i = 1$. Note that this is different from the convex hull of the function \tilde{s} .

attention to \mathcal{F} instead of its convex hull. Then, the proof considers a relaxed problem over $\tilde{s} \in \{\tilde{s} \mid \tilde{s} = \phi \circ s, s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})\}$, instead of $\tilde{s} \in \mathcal{F}$, and then verifies that the solution satisfies the original constraint $\tilde{s} \in \mathcal{F}$.²⁷

Proposition 7. *Assume ϕ is continuously differentiable and strictly increasing.*

1. *If ϕ is convex,*

- *The revenue-maximizing mechanism fully separates participants (and excludes $\theta < J^{-1}(0)$) if $J(\theta)$ is increasing and $v(\theta) = \alpha\theta$.*
- *Holding exclusion fixed, as the status profile becomes finer, consumer surplus increases if F satisfies DFR.*
- *Consumer surplus is maximized by full separation under DFR.*
- *The social welfare-maximizing mechanism fully separates participants if $J_\lambda(\theta)$ is increasing and $v(\theta) = \alpha\theta$.*

2. *If ϕ is concave,*

- (a) *The revenue-maximizing mechanism fully separates participants if $J(\theta)\phi'(F(\theta))$ is increasing.*
- (b) *Holding exclusion fixed, as the status profile becomes finer, consumer surplus decreases if F satisfies IFR.*
- (c) *Consumer surplus is maximized by total pooling under IFR, and maximized by full separation if $\frac{1-F(\theta)}{f(\theta)}\phi'(F(\theta))$ is increasing.*
- (d) *The social welfare-maximizing mechanism fully separates participants if $J_\lambda(\theta)\phi'(F(\theta))$ is increasing.*

Remark 5. The conditions also applies to the specification where the status remains linear (i.e., $S(x, G) = \frac{1}{2}(G^-(x) + G(x))$), but the agent's payoff is strictly increasing and convex or concave in status—i.e., $u(p, s, \theta) = \theta\phi(s) - p + v(\theta)$.

When ϕ is convex, the results for full separation follow from the Fan-Lorentz Theorem because $z(\theta)\phi(s)$ is convex in s and supermodular in (s, θ) for both $z \in \{J, J_\lambda, \frac{1-F(\theta)}{f(\theta)}\}$ when $z(\theta)$ is positive and increasing.

²⁷Consequently, the conditions in the proposition also applies to the specification where status remains linear (i.e., $S(x, G) = \frac{1}{2}(G^-(x) + G(x))$), but the agent's payoff is nonlinear in status: $u(p, s, \theta) = \theta\phi(s) - p + v(\theta)$.

When ϕ is concave, the results under IFR also follow from Fan-Lorentz because $\frac{1-F(\theta)}{f(\theta)}\phi(s)$ is concave in s and submodular in (s, θ) . In other cases, when the condition involves $\phi'(F(\theta))$, the proof uses the optimal control method. Because $J(\theta)\phi'(F(\theta))$ may be decreasing if $J(\theta)$ is increasing, the revenue-maximizing mechanism may involve pooling even under Myerson's regularity condition.

5.1.2 Signaling

Consider an alternative specification where status arises from signaling concerns. For example, consumers purchase conspicuous goods x to obtain social status valued at $\mathbf{E}[\theta|x]$ (Rayo, 2013). Under this specification, the status induced by a positional good allocation $\chi: \Theta \times \Omega \rightarrow \tilde{X}$ is given by

$$S(x, G_\chi) = \mathbf{E}[\theta \mid \chi(\theta, \omega) = x].$$

An important implication of this specification is that, even if a consumer does not purchase the good (i.e., $\sigma(\theta) = 0$), he still obtains a baseline status $\underline{s} = \mathbf{E}[\theta \mid \chi(\theta) = 0]$ rather than zero as in the main specification.

This feature has two implications. First, the feasibility condition in Theorem 1 becomes analogous to the case without exclusion (see Remark 2): an incentive-compatible (and hence increasing) $s(\theta)$ is feasible if and only if $s \in \text{MPS}(\theta)$ in quantile space, that is,

$$\int_{\theta}^{\bar{\theta}} s(t) dF(t) \leq \int_{\theta}^{\bar{\theta}} t dF(t) \quad \text{for all } \theta \in [0, \bar{\theta}] \quad (14)$$

with equality at $\theta = 0$. In terms of status profiles, being excluded is equivalent to being pooled in a single bottom tier, although the buyer obtains an intrinsic value $v(\theta)$ in the latter case.

Moreover, the buyer's outside option becomes $\theta_{\underline{s}}$ instead of zero. When the seller excludes types $\theta < \theta_0$, the baseline status is $\underline{s} = \mathbf{E}[\theta \mid \theta < \theta_0]$. If the intrinsic value $v(\theta) = 0$, instead of excluding types $\theta \in [0, \theta_0)$, the seller can achieve the same revenue by also offering them a free lowest-level product that pools them in a single bottom tier, which replicates the same status level as the baseline \underline{s} without affecting the incentives of higher types. Hence, it is without loss of optimality to assume no exclusion; in other words, exclusion has no benefit through the status channel, in contrast to the main specification.²⁸

²⁸Of course, if the intrinsic value $v(\theta) > 0$, then exclusion may increase revenue through the other (standard) channel.

Consequently, if $v(\theta) = 0$, the main results still hold except that there is no exclusion, analogous to the case where exclusion is assumed to be impossible (see Appendix A): the revenue-maximizing mechanism fully separates all types if and only if $J(\theta)$ is increasing, while consumer surplus decreases (increases) as the seller offers more levels if F has an increasing (decreasing) failure rate. In particular, if $J(\theta)$ is strictly increasing on the bottom of the support, exclusion strictly reduces revenue.

5.1.3 Varying Positional Concerns about Same-Level Consumers

Now consider a more general specification à la Hopkins and Kornienko (2004):

$$S(x, G(\cdot)) = \gamma G(x) + (1 - \gamma)G^-(x),$$

where $\gamma \in [0, 1]$ measures the intensity of concern about the mass of consumers at the same level. The benchmark case $\gamma = 1/2$ ensures that, for a given level of exclusion, the participants' expected total status is invariant to the number of status levels. If $\gamma < 1/2$ ($\gamma > 1/2$), consumers discount same-level consumers more (less) heavily, so pooling decreases (increases) expected aggregate status and the feasibility theorem fails in general. The linear case in Immorlica, Stoddard, and Syrgkanis (2015) corresponds to $\gamma = 0$, where consumers discount same-level consumers entirely and derive utility from the number of consumers at strictly lower levels. At the other extreme, Frank (1985b) assumes $\gamma = 1$, where consumers do not distinguish same-level consumers from lower-level ones.

Although the feasibility condition in Theorem 1 does not hold when $\gamma \neq 1/2$, the main results for the baseline case $\gamma = 1/2$ provide a useful benchmark. For revenue maximization, the first part (when J is increasing) of Proposition 2 still holds if $\gamma \in [0, 1/2]$. Intuitively, when $\gamma < 1/2$, pooling lowers the average status of the pooled types, reinforcing the majorization argument against pooling.

For consumer surplus maximization under nonnegative prices, total pooling without exclusion maximizes consumer surplus under IFR if $\gamma \in [1/2, 1]$, while full separation without exclusion maximizes consumer surplus under DFR if $\gamma \in [0, 1/2]$. Intuitively, $\gamma > 1/2$ raises the average status of pooled types, making pooling more attractive, while $\gamma < 1/2$ lowers average status of pooled types, making pooling less attractive. In particular, full separation is always optimal if $\gamma = 0$, while total pooling is always optimal if $\gamma = 1$.

5.2 Intrinsic Quality

In the baseline mode, to abstract from quality differentiation, I assume that positional goods have the same intrinsic quality across tiers and thus deliver the same intrinsic value $v(\theta)$. In many applications, however, a higher-tier positional good, such as a larger car or a higher boarding class, also comes with a higher intrinsic quality.

In this subsection, I assume that the tier x is not only ordinal. Instead, a good $x \in \tilde{X} = \mathbb{R}_+$ has intrinsic quality equal to x , which delivers intrinsic value $v(x, \theta) = \theta x$ and incurs a production cost $c(x)$, à la [Mussa and Rosen \(1978\)](#). Assume that $c(x)$ is continuously differentiable, strictly increasing, strictly convex, and satisfies $c(0) = c'(0) = 0$ and $\lim_{x \rightarrow \infty} c'(x) = \infty$. Because $v(0, \theta) = 0$ and $c(0) = 0$, it is still without loss of generality to denote the outside option of not buying by $x = 0$.

As before, consider a direct mechanism $(\chi(\theta, \omega), p(\theta))$ consisting of an allocation rule $\chi: \Theta \times \Omega \rightarrow \mathbb{R}_+$ and a payment schedule $p: \Theta \rightarrow \mathbb{R}$. The participation decision is incorporated into the allocation χ , as $\chi(\theta, \omega) = 0$ is equivalent to opting out. Let $x(\theta) = \mathbf{E}_\omega[\chi(\theta, \omega)]$ denote the interim allocation. Consider the direct mechanism $(s(\theta), x(\theta), p(\theta))$, where $s(\theta)$ is the status profile induced by χ . Thus, the buyer's utility is $U(\theta) = \theta(s(\theta) + x(\theta)) - p(\theta)$. By convention, the opt-out decision is $\chi(\theta) = 0$ (which induces $s(\theta) = 0$) and $p(\theta) = 0$. The following lemma on incentive compatibility replaces Lemma 1.

Lemma 2. *A direct mechanism $(s(\theta), x(\theta), p(\theta))$ is incentive-compatible if and only if*

- $s(\theta) + x(\theta)$ is increasing;
- $U(\theta) = U(0) + \int_0^\theta (s(t) + x(t)) dt$ for all $\theta \in [0, \bar{\theta}]$.

In addition to incentive compatibility, the mechanism must satisfy the compatibility constraint that $s(\theta)$ and $x(\theta)$ must be generated by the same allocation χ . Say s and x are *compatible* if $s(\theta) = \mathbf{E}_\omega[S(\chi(\theta, \omega), G_\chi)]$ and $x(\theta) = \mathbf{E}_\omega[\chi(\theta, \omega)]$ for some allocation $\chi: \Theta \times \Omega \rightarrow \mathbb{R}_+$. Using the same arguments as before, the revenue maximization problem is

$$\max_{\chi: \Theta \times \Omega \rightarrow \mathbb{R}_+} \int_0^{\bar{\theta}} (J(\theta)(s(\theta) + x(\theta)) - \mathbf{E}_\omega[c(\chi(\theta, \omega))]) dF(\theta) \quad (15)$$

subject to monotonicity and compatibility constraints. Under the regularity condition that $J(\theta)$ is increasing, there is no loss of optimality in restricting attention to deterministic increasing allocations $\chi: \Theta \rightarrow \mathbb{R}_+$ (see Lemma B.5).

In general, because of the compatibility constraint, the optimization over status s and quality x cannot be separated. However, in this case, the separate optimizers over s and x happen to be compatible, as they can be generated by the same deterministic allocation χ .

Proposition 8. Assume $J(\theta)$ is increasing. The profit-maximizing mechanism excludes $\theta < \hat{\theta}_0 = \inf\{\theta \in [0, \bar{\theta}] : J(\theta) > 0\}$ and assigns $\chi^*(\theta) = c'^{-1}(J(\theta)) \cdot \mathbf{1}_{\theta \geq \hat{\theta}_0}$, which induces $x^*(\theta) = \chi^*(\theta)$ and

$$s^*(\theta) = \begin{cases} 0, & \text{if } \theta < \hat{\theta}_0 \\ \frac{F(\underline{\theta}_i) + F(\bar{\theta}_i)}{2}, & \text{if } \theta \in [\underline{\theta}_i, \bar{\theta}_i] \\ F(\theta), & \text{otherwise} \end{cases}$$

where $[\underline{\theta}_i, \bar{\theta}_i]$ are the intervals where $J(\theta) > 0$ is constant.

The optimal status mechanism excludes types below $\hat{\theta}_0$, for which the virtual valuation $J(\theta)$ is negative. For types above $\hat{\theta}_0$, if $J(\theta)$ is strictly increasing, the optimal mechanism separates all types and induces $s^*(\theta) = F(\theta)$; if $J(\theta)$ is constant, the optimal mechanism pools types with the same virtual valuation and induces the same status for them. The proposition implies that the profit-maximizing mechanism is robust to intrinsic quality, or equivalently, that the quality differentiation result in [Mussa and Rosen \(1978\)](#) is robust to positional externalities.

By attaching status value, the seller can extract $\int_{\hat{\theta}_0}^{\bar{\theta}} J(\theta) s^*(\theta) dF(\theta) > 0$ additional revenue. For the uniform distribution on $[0, 1]$ and quadratic cost $c(x) = x^2/2$, profit with status value is \$0.29 compared to \$0.08 with pure intrinsic value. This may explain why luxury companies invest heavily in cultivating status value.

5.3 Screening with Excessive Waiting Time (Ordeals)

Priority service is an example of positional goods that arise from capacity constraints. As shown in [Example 1](#), the baseline model directly applies after defining the status by $s = 1 - t$, where t is the waiting time. In this subsection, I consider an extension where the service provider can delay service and prolong waiting time to $t = 1 - s > 1$.

When the seller can delay service, the range of status profiles becomes $(-\infty, 1]$. Incentive compatibility still requires the status profile $s(\theta)$ to be increasing, and the set of feasible status profiles is the lower set of the original feasible set, that is, $\{s \mid s \leq \hat{s} \text{ for some } \hat{s} \in \text{MPS}_0(F)\}$

Can the seller benefit from delaying service? The excessive waiting time $t > 1$ can be viewed as an ordeal to screen low types. Intuitively, instead of excluding low types, the seller can serve them with excessive waiting time, thereby extracting some revenue from them while preventing higher types from mimicking them. Indeed, as shown in the following proposition, it is optimal to assign $t(\theta) = 1 + v'(\theta)$ (i.e., $s(\theta) = -v'(\theta)$) to buyers with negative virtual valuation (i.e., $J(\theta) < 0$). Moreover, under IFR, this increases

the mass of consumers who receive zero payoff, effectively increasing exclusion and reducing consumer surplus.

Proposition 9. *Assume $J(\theta)$ is increasing and that delaying service is feasible. Let $\hat{\theta}_0 = \inf\{\theta : J(\theta) > 0\}$. Then, the revenue-maximizing mechanism is given by*

$$s^*(\theta) = \begin{cases} F(\theta), & \text{if } \theta \geq \hat{\theta}_0 \\ -v'(\theta), & \text{if } \theta < \hat{\theta}_0 \end{cases} \text{ and } p^*(\theta) = \begin{cases} \hat{\theta}_0 F(\hat{\theta}_0) + \int_{\hat{\theta}_0}^{\theta} z dF(z) + v(\hat{\theta}_0), & \text{if } \theta \geq \hat{\theta}_0 \\ v(\theta) - \theta v'(\theta), & \text{if } \theta < \hat{\theta}_0. \end{cases}$$

Allowing the seller to delay service strictly increases revenue and decreases consumer surplus if $v(\theta) > \theta v'(\theta)$ (i.e., if v is strictly concave or $v(0) > 0$) and $f(0) < \infty$.

The waiting time is $t^*(\theta) = 1 - s^*(\theta)$. For types $\theta < \hat{\theta}_0^*$, the allocation $s^*(\theta) = -v'(\theta)$ is increasing by the assumption $v''(\theta) \leq 0$. It can also be induced by randomization between pooling and separation after pausing the service, as illustrated in the following example.

Example 4 (Delaying Service). Consider the payoff $u(p, s, \theta) = \tilde{v}(\theta) - \theta t - p$, where $t = 1 - s$ is the waiting time. Assume $\theta \sim \text{Unif}[0, 1]$ and $\tilde{v}(\theta) = v_0 + (1 + \alpha)\theta$ (so that $v(\theta) = v_0 + \alpha\theta$), where $v_0, \alpha > 0$. Then, the revenue-maximizing mechanism is

$$t^*(\theta) = \begin{cases} 1 - \theta, & \text{if } \theta \geq 1/2 \\ 1 + \alpha, & \text{if } \theta < 1/2 \end{cases} \text{ and } p^*(\theta) = \begin{cases} \theta^2/2 + \alpha/2 + 1/8 + v_0, & \text{if } \theta \geq 1/2 \\ v_0, & \text{if } \theta < 1/2. \end{cases}$$

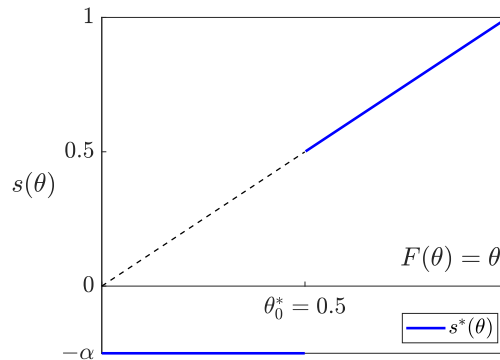


Figure 3: Delaying Service

Instead of excluding types below $1/2$, the seller serves them with excessive waiting time $t^*(\theta) = 1 + \alpha$ and charges a price $p^*(\theta) = v_0$. The excessive waiting time $1 + \alpha$ can be implemented by pausing service for $\alpha + 1/4$ units of time before serving types $\theta < 1/2$ in random order (by pooling them into the same priority level).

The insight that excessive waiting time serves as a substitute for exclusion also applies to the case where exclusion is impossible, as the lowest-level good or service is free (see Appendix A). In this case, excessive waiting time diminishes the value of the free service and effectively plays the role of exclusion, thereby increasing the seller's revenue. In particular, by offering a priority service and a free regular service, the seller extracts more revenue as she increases the waiting time for the free regular service.

5.4 When Higher Types Obtain Lower Utility

Now I consider the case where $v(\theta) \geq 0$ and $v'(\theta) \leq -1$, so that higher types obtain lower total utility from purchasing the good, accounting for both status and intrinsic value. This arises naturally in Example 1: if the good has homogeneous value across agents, higher (cost) types derive lower net utility after accounting for waiting costs.²⁹ Therefore, the relevant incentive pressure is upward: agents must be prevented from *overreporting* in order to obtain shorter waiting times, as in procurement auctions.

Formally, since $s(\theta) \leq 1$, when $v'(\theta) \leq -1$, the envelope condition implies

$$U'(\theta) = s(\theta) + v'(\theta) \leq 0$$

for all participating types θ (for which $\sigma(\theta) = 1$). Thus, in contrast to the main specification, higher types receive *lower* information rents, so agents must be prevented from overreporting, and the local IC constraint binds from below rather than from above. Moreover, it is now *high* types that are potentially excluded: there exists a cutoff $\theta_0 \in [0, \bar{\theta}]$ such that a buyer participates (i.e., $\sigma(\theta) = 1$) for all $\theta < \theta_0$ and does not participate (i.e., $\sigma(\theta) = 0$) for all $\theta > \theta_0$.

Under full separation, the status profile is given by $s(\theta) = F(\theta) + (1 - F(\theta_0))$. The feasibility condition in terms of majorization continues to hold, up to a shift, for all participants $\theta \in [0, \theta_0]$. Formally, an incentive-compatible s is feasible if and only if there exists $\theta_0 \in [0, \bar{\theta}]$ such that $s - (1 - F(\theta_0)) \in \text{MPS}(F \cdot \mathbf{1}_{[0, \theta_0]})$ (in quantile space), and $s(\theta) = 0$ for all $\theta \in (\theta_0, \bar{\theta}]$.

The following proposition shows the results for the case where $v'(\theta) \leq -1$.

Proposition 10. *The following results hold if $v'(\theta) \leq -1$:*

(i) *Assume $L(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$ is increasing. Then,*

(a) *The revenue is increasing in the number of positional good levels.*

²⁹In Example 1, if the good has the same value v_0 to everyone, then the payoff $u(p, s, \theta) = v_0 - \theta t - p = (v_0 - \theta) + \theta s - p$ (where $s = 1 - t$) satisfies $v'(\theta) = -1$.

- (b) *The revenue-maximizing mechanism excludes $\theta > \theta_0^*$ and fully separates $\theta \leq \theta_0^*$, where the optimal cutoff is $\theta_0^* = \arg \max_{\theta_0} \int_0^{\theta_0} L(\theta)F(\theta) dF(\theta) + [v(\theta_0) + \theta_0(1 - F(\theta_0))]F(\theta_0)$.*
- (c) *If $v(\theta)F(\theta)$ is increasing on $[0, \bar{\theta}]$, then no exclusion is optimal (i.e., $\theta_0^* = \bar{\theta}$).*
- (ii) *The seller can obtain at least half the maximum revenue by selling a single good.*
- (iii) *Under the nonnegative price constraint, consumer surplus is maximized by total pooling (full separation) without exclusion if the reverse failure rate $\frac{f(\theta)}{F(\theta)}$ is decreasing (increasing).*
- (iv) *Delaying service cannot increase the seller's revenue.*

Parts (i)–(iii) are qualitatively similar to those in the main specification, with the integrand $J(\theta)$ replaced by $L(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$, and that the revenue-maximizing mechanism excludes high types instead of low types (because $U'(\theta) \leq 0$). For part (iv), when the seller assigns $t > 1$ (or $s < 0$), she must assign it to low types first because s is increasing. However, since agents now have incentives to *overreport* (because $U'(\theta) \leq 0$), assigning negative status to *low* types does not mitigate agents' incentives to overreport and therefore cannot increase the seller's revenue.

6 Conclusion

In this paper, I study the allocation of positional goods using a mechanism design approach. Many goods are positional because consumers care about their relative consumption, either for psychological (such as luxury goods) or physical reasons (such as priority services). Thus, allocating positional goods generates externalities on others: improving one consumer's position necessarily worsens another's, and the value of the good diminishes as more consumers buy it. Because of these externalities, the optimal mechanism in terms of revenue and consumer welfare are different from allocating ordinary goods.

I first characterize the revenue-maximizing mechanism. When the distribution satisfies Myerson's regularity condition, the revenue-maximizing mechanism can be implemented by an all-pay auction with a reserve price, in which buyers who pay more receive higher positions. The seller can guarantee at least half the maximal revenue by offering a single tier. To evaluate consumer welfare when regulations or exogenous restrictions prevent the seller from offering the revenue-maximizing mechanism, I also

provide comparative statics on the welfare effects of offering more levels of positional goods and excluding consumers. Furthermore, I characterize the consumer-optimal mechanism with and without subsidies, which depends on the failure rate of the type distribution.

The results further illuminate the tradeoffs of educational disarmament. To maximize aggregate effort, meritocratic competition with exclusion is usually optimal. For student welfare, however, meritocracy is harmful under common thin-tailed ability distributions, consistent with the common view that meritocratic competition generates a rat race of wasteful effort. Thus, this result suggests the need for disarmament policies such as admission lotteries or coarser performance rankings. When the distribution is heavy-tailed, however, meritocracy raises student welfare in the aggregate, as high-ability students benefit substantially from separation. From a social welfare perspective, when effort is sufficiently productive, the effort gains from meritocracy render disarmament unnecessary. Regardless of the distribution, the student-optimal mechanism does not involve exclusion.

Several extensions merit further investigation. Competition among sellers of positional goods remains unexplored. In the education context, this can be interpreted as competition between different tracks leading to different status levels (e.g., vocational versus academic education). The problem of sellers competing strategically in mechanisms remains open.

Appendix A Optimal Mechanisms without Exclusion

In this section, I study the optimal mechanisms when exclusion is impossible and the lowest-level positional good is available for free.³⁰ For example, in [Moldovanu, Sela, and Shi \(2007\)](#), agents cannot be excluded and are guaranteed at least the lowest status in the organization; in the benchmark model of [Gershkov and Winter \(2023\)](#), buyers can use the regular (non-priority) service for free. The free lowest-level position, rather than opting out, thus serves as the buyer’s effective outside option. Moreover, as noted in Remark 2, the feasibility condition becomes $s \in \text{MPS}(F)$.

Based on the feasibility condition, I first characterize the revenue-maximizing mechanism (Proposition A.1). In particular, I provide necessary and sufficient conditions under which the revenue-maximizing mechanism is fully separating. I also establish a 2-approximation result: selling a premium tier alongside the free tier guarantees at least half the maximum revenue (Proposition A.2). Then, I study consumer surplus and provide conditions under which it is decreasing or increasing in the number of positional good levels offered by the seller (Proposition A.3). I also characterize the necessary and sufficient condition under which introducing a premium tier alongside the free tier increases consumer welfare (Proposition A.4) and under which introducing premium tiers hurts every consumer (Proposition A.5).

A.1 Revenue Maximization

The revenue maximization problem is given by

$$\max_{s(\theta), p(\theta)} \int_0^{\bar{\theta}} p(\theta) dF(\theta) \quad (\text{A.1})$$

subject to the following constraints on $\theta \in [0, \bar{\theta}]$

$$U(\theta) - v(\theta) = \theta s(\theta) - p(\theta) \geq \theta \underline{s} \quad (\text{IR}) \quad (\text{A.2})$$

$$U(\theta) - v(\theta) = U(0) - v(0) + \int_0^\theta s(t) dt \quad (\text{IC}) \quad (\text{A.3})$$

$$s(\theta) \text{ is increasing} \quad (\text{A.4})$$

$$s \in \text{MPS}(F) \quad (\text{MPS}) \quad (\text{A.5})$$

³⁰This section subsumes results in my note “A Mechanism Design Approach to ‘Gainers and Losers in Priority Services.’”

where $\underline{s} = \min\{s(\theta)\}$ denotes the lowest status, which is endogenously determined by the status allocation. The (IR) constraint, which is equivalent to $\tilde{U}(\theta) \equiv U(\theta) - v(\theta) - \theta \underline{s} \geq 0$, can be reduced to $U(0) - v(0) \geq 0$ because $U'(\theta) - v'(\theta) - \underline{s} = s(\theta) - \underline{s} \geq 0$.

By standard arguments, the expected revenue is given by

$$R = \int_0^{\bar{\theta}} p(\theta) dF(\theta) = \int_0^{\bar{\theta}} \left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) s(\theta) dF(\theta) - (U(0) - v(0)). \quad (\text{A.6})$$

By (IR), it is optimal to set $U(0) = v(0)$. The revenue maximization problem is equivalent to

$$\max_{s \in \text{MPS}(F)} \int_0^{\bar{\theta}} J(\theta) s(\theta) dF(\theta). \quad (\text{A.7})$$

Theorem 4 (Fan-Lorentz) in KMS implies the following result.

Proposition A.1. *Offering more positional good levels always (strictly) increases the seller's revenue if and only if $J(\theta)$ is (strictly) increasing. Thus, full separation maximizes revenue if and only if $J(\theta)$ is increasing.*

If $J(\theta)$ is not monotonic, define $\tilde{J}(\theta) = \int_0^\theta J(t) dF(t)$ and let $K(\tau) = \text{conv } \tilde{J}(F^{-1}(\tau))$ denote the convex hull of \tilde{J} in quantile space. Then, the revenue-maximizing mechanism pools types into the same positional good level if K is affine and separates types if $K \circ F = \tilde{J}$.

Remark A.1. GW's Proposition 8 provides a sufficient condition for the seller's revenue to be strictly increasing in the number of priority classes: F satisfies the IFR property.

Remark A.2. Effort maximization in MSS's model is the same as revenue maximization because they assume linear effort costs (and no exclusion). In Theorem 4, they also provide a sufficient condition for full separation to be optimal: F satisfies the IFR property.

Infinitely many positional good levels can be implemented by an all-pay auction, in which the more money a consumer pays, the higher status he receives.

Now consider the approximation of selling a single positional good in addition to a free low-level position, as in Example 3.

Proposition A.2. *The seller can obtain at least half the maximum revenue by selling a single positional good in addition to a free low-level position.*

Proof. Denote by p the price of the high-level position. The cutoff type $\theta(p)$ indifferent between two levels is given by

$$\theta(p) \frac{1 + F(\theta(p))}{2} - p = \theta(p) \frac{F(\theta(p))}{2} \iff \theta(p) = 2p.$$

Let R_2 denote the seller's maximum revenue from offering one positional good in addition to a free low-level position, which is given by

$$R_2 = \max_p p(1 - F(2p)) = \frac{1}{2} \max_p p(1 - F(p)).$$

Consider the auxiliary problem of selling an indivisible good to one buyer, in which a standard extreme-point argument implies a posted-price mechanism is optimal (see, e.g., [Börgers, 2015](#), Proposition 2.5). In other words, let $\mathcal{M} = \{q: [0, \bar{\theta}] \rightarrow [0, 1] \mid q \text{ increasing}\}$ denote the set of incentive-compatible allocations, then

$$\max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) = \max_p p(1 - F(p)).$$

Because $\text{MPS}(F) \subseteq \mathcal{M}$, we have

$$\max R = \max_{s \in \text{MPS}(F)} \int_0^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) < \max_{x \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)x(\theta) dF(\theta) = \max_p p(1 - F(p)) = 2R_2.$$

The inequality is strict because any maximizer \tilde{q}^* of the auxiliary problem satisfies $\tilde{q}^*(\theta) = 1$ on $[\tilde{p}^*, \bar{\theta}]$ for some $\tilde{p}^* < \bar{\theta}$, and thus $\tilde{q}^* \notin \text{MPS}(F)$. \square

This proposition extends Proposition 3 to the case where exclusion is impossible.

Graphical Illustration. Figure 4 plots the *revenue curve* $R(\tau) = (1 - \tau)F^{-1}(\tau)$ in quantile space, where $\tau = F(\theta)$. Assume $v(\theta) = 0$ for simplicity. Similar to Section 3.2, (i) the area under the revenue curve (*blue* area in the left panel) represents the revenue from the revenue-maximizing mechanism—i.e., $s^*(\theta) = F(\theta)$, (ii) the *red* triangle in the right panel represents the revenue from having two levels of positional goods—a higher level offered at price $F(\theta_0^*)$ in addition to a lower level offered for free, and (iii) the entire box of either panel (which has an area of $R(\tau_0^*)$) represents the maximum revenue from selling nonpositional goods in the auxiliary problem. It is straightforward that the red area (ii) is larger than half of the entire box and thus strictly larger than half of the blue area (i).

The approximation performs well for many common distributions, as shown in the following examples (with $v(\theta) = 0$).

Example (Exponential distribution). If $F(\theta) = 1 - \exp(-\lambda\theta)$ where $\lambda > 0$, offering a single good above the free lowest level can obtain 73.6% of the maximum revenue.

Example (Uniform distribution). If $F(\theta) = \theta$ on $[0, 1]$, offering a single good above the free lowest level can obtain 75% of the maximum revenue.

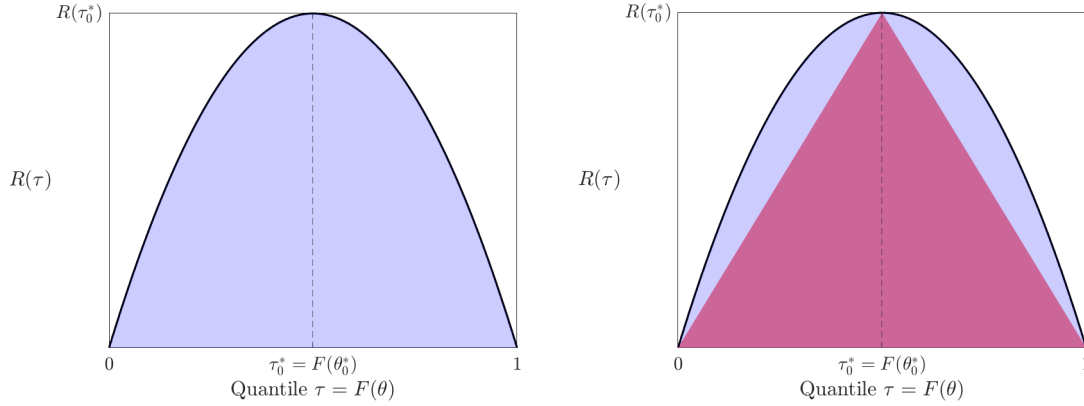


Figure 4: Revenue curve $R(\tau)$ for the uniform distribution

Example (Power distribution). If $F(\theta) = \theta^\beta$ on $[0, 1]$ where $\beta > 0$, offering a single good above the free lowest level can obtain

$$\frac{R_2}{\max R} = \begin{cases} \frac{(1 + 2\beta)}{2\beta(1 + \beta)^{1/\beta}} > 72.1\%, & \text{if } \beta \geq 1 \\ \frac{1 + 2\beta}{(1 + \beta)^{1/\beta} [2\beta + (1 - \beta)^{2+1/\beta}]} > \frac{3}{4}, & \text{if } 0 < \beta < 1 \end{cases}$$

of the maximum revenue. The ratio approaches 1 as $\beta \rightarrow \infty$ (i.e., F is sufficiently convex) or $\beta \rightarrow 0$ (i.e., F is sufficiently concave).

A.2 Consumer Surplus

Consumer surplus is given by

$$W = \int_0^{\bar{\theta}} U(\theta) dF(\theta) = \int_0^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} \right) s(\theta) dF(\theta) + \mathbf{E}[v(\theta)] + U(0) - v(0). \quad (\text{A.8})$$

Proposition A.3. For any increasing allocations $s, \hat{s}: [0, \bar{\theta}] \rightarrow [0, 1]$ such that $\hat{s} \in \text{MPS}(s)$,

- (i) \hat{s} results in a higher consumer surplus if F satisfies IFR;
- (ii) \hat{s} results in a lower consumer surplus if and only if F satisfies DFR.

Therefore, consumer surplus decreases (increases) as the seller offers more levels of positional goods if F satisfies IFR (DFR).

I focus on the case where the price (or effort) is nonnegative, which pin down $p(0) = v(0) - U(0) = 0$.

Corollary A.3.1. *Consumer surplus is the highest when the seller offers*

(i) *one position level (i.e., $s(\theta) = 1/2$) if F satisfies IFR;*

(ii) *full separation (i.e., $s(\theta) = F(\theta)$) if and only if F satisfies DFR.*

In the former case, consumer surplus is $\mathbf{E}[v(\theta) + \theta/2]$.

Remark A.3. GW's Proposition 7 shows the sufficiency of IFR property ($\frac{1-F(\theta)}{f(\theta)}$ is decreasing) for no priority service to be consumer surplus-maximizing.

Remark A.4. If both $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$ and $\frac{1-F(\theta)}{f(\theta)}$ are increasing (e.g., exponential and Pareto distributions), a finer partition (in terms of majorization) can increase both the seller's revenue and consumer surplus. Thus, full separation maximizes both the revenue and consumer surplus.

Proposition A.4. *Consumer surplus is the highest when the seller offers a single (free) level if and only if*

$$\int_0^\theta \left(\frac{1-F(t)}{f(t)} - \mathbf{E}[\theta] \right) dF(t) \geq 0 \quad \text{for all } \theta \in [0, \bar{\theta}].$$

Proof. Define $H(\theta) = \int_0^\theta \frac{1-F(t)}{f(t)} dF(t)$. Then, the condition in the proposition is equivalent to $H(\theta) \geq H(\bar{\theta})F(\theta) = \mathbf{E}[\theta]F(\theta)$ (i.e., $H(\theta)$ lies above the line connecting $H(0) = 0$ and $H(\bar{\theta}) = \mathbf{E}[\theta]$ in quantile space). Therefore, by Proposition 2 in KMS, this condition is necessary and sufficient for total pooling to be welfare-maximizing. \square

Remark A.5. A sufficient condition is that $\frac{1-F(\theta)}{f(\theta)}$ single-crosses $\mathbf{E}[\theta]$ from above. An even stronger sufficient condition is the IFR property, i.e., $\frac{f(\theta)}{1-F(\theta)}$ is increasing.

Remark A.6. This condition is necessary and sufficient for customers' welfare to be higher when a free level is offered than when *any* $k > 1$ levels are offered. By contrast, GW's Proposition 1 provides a sufficient condition for customers' welfare to be higher when a free level is offered than when one additional level is introduced.

Moreover, fix a type $\theta \in [0, \bar{\theta}]$, maximizing his utility $U(\theta)$ subject to constraints (A.2)–(A.5) is equivalent to

$$\max_{s \in \text{MPS}(F)} \int_0^\theta \frac{s(t)}{f(t)} dF(t) + v(\theta) \tag{A.9}$$

The following proposition provides a necessary and sufficient condition for total pooling to maximize $U(\theta)$ for all $\theta \in [0, \bar{\theta}]$.

Proposition A.5. *Every consumer's utility is the highest when the seller offers one position level if and only if $F(\theta) \leq \theta/\bar{\theta}$ for all $\theta \in [0, \bar{\theta}]$ (i.e., F first-order stochastic dominates the uniform distribution). A sufficient condition is that $f(\theta)$ is increasing.*

Proof sketch. $U(\theta) = \int_0^\theta \frac{s(t)}{f(t)} dF(t) + v(\theta)$. If f is increasing, then $\frac{1}{f(\theta)}$ is decreasing, so $s = 1/2$ maximizes $U(\theta)$.

Note that $\int_0^\theta \frac{1}{f(t)} dF(t) = \theta$, and the condition $F(\theta) \leq \theta/\bar{\theta}$ is equivalent to $\int_0^\theta (\frac{1}{f(t)} - \frac{1}{\bar{\theta}}) dF(t) \geq 0$ for all $\theta \in [0, \bar{\theta}]$. By the same argument as in the proof of Proposition A.4, this condition is necessary and sufficient for $s(\theta) = 1/2$ to maximize $U(\theta)$ for all $\theta \in [0, \bar{\theta}]$. \square

Remark A.7. Proposition A.5 provides the necessary and sufficient condition for all consumers to be worse off after *any* $k > 1$ levels of positional goods are offered than if one level is offered (for free). GW's Proposition 2 shows that if $F(c) \leq c/\bar{c}$, all consumers are worse off after the introduction of *one* additional level of priority service.

Appendix B Proofs

B.1 Proof of Theorem 1

Proof. (\implies) Suppose that an increasing s is induced by some allocation $\chi: \Theta \times \Omega \rightarrow [0, 1]$. Because $S(0, G) = 0$, when $\chi(\theta, \omega) = 0$ for all $\omega \in \Omega$, we have $s(\theta) = 0$. Since s is increasing, there exists a participation cutoff $\theta_0 \in [0, \bar{\theta}]$ such that $s(\theta) = 0$ for all $\theta < \theta_0$. For all $\theta \in [\theta_0, \bar{\theta}]$, as switching to full separation $s(\theta) = F(\theta)$ increases status for higher types and decreases status for lower types, we have that

$$\mathbf{E}[s(\theta) \mid \theta \geq t] \leq \mathbf{E}[F(\theta) \mid \theta \geq t].$$

Thus,

$$\frac{1}{1 - F(t)} \int_t^{\bar{\theta}} s(\theta) dF(\theta) \leq \frac{1}{1 - F(t)} \int_t^{\bar{\theta}} F(\theta) dF(\theta), \quad \forall t \in [\theta_0, \bar{\theta}]$$

Since each pooling interval preserves total status, summing over all intervals gives

$$\int_{\theta_0}^{\bar{\theta}} s(\theta) dF(\theta) = \int_{\theta_0}^{\bar{\theta}} F(\theta) dF(\theta).$$

(\impliedby) Suppose there exists $\theta_0 \in [0, \bar{\theta}]$ such that $s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$ for all $\theta \geq \theta_0$ and $s(\theta) = 0$ for all $\theta < \theta_0$. I show that s is feasible.

First, by Theorem 1 in KMS, the extreme point of $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$ such that $s(\theta) = 0$ for all $\theta < \theta_0$ is characterized by a collection of intervals $[\underline{\theta}_i, \bar{\theta}_i] \subseteq [\theta_0, \bar{\theta}]$, indexed by $i \in I$, such that

$$\tilde{s}(\theta) = \frac{1}{F(\bar{\theta}_i) - F(\underline{\theta}_i)} \int_{\underline{\theta}_i}^{\bar{\theta}_i} F(\tilde{\theta}) dF(\tilde{\theta}) = \frac{F(\underline{\theta}_i) + F(\bar{\theta}_i)}{2}, \quad \text{if } \theta \in [\underline{\theta}_i, \bar{\theta}_i],$$

and $\tilde{s}(\theta) = F(\theta)$ if $\theta \notin \bigcup_{i \in I} [\underline{\theta}_i, \bar{\theta}_i]$.

Each such extreme point is implementable by an increasing deterministic allocation $\chi: \Theta \rightarrow \mathbb{R}_+$. For all $\theta < \theta_0$, assign the outside option $\chi(\theta) = 0$. For all $\theta \geq \theta_0$, choose a (weakly) increasing χ such that: on each pooling interval $[\underline{\theta}_i, \bar{\theta}_i]$, $\chi(\theta) = x_i > 0$; outside the pooling intervals, assign a strictly increasing $\chi(\theta)$. Under the status specification

$$S(x, G) = \begin{cases} \frac{G^-(x) + G(x)}{2} & \text{if } x > 0, \\ 0 & \text{if } x = 0, \end{cases}$$

this deterministic allocation induces the extreme point \tilde{s} . Indeed, if χ is strictly increasing at θ , then $G(\theta) = G^-(\theta) = F(\theta)$. If $\theta \in [\underline{\theta}_i, \bar{\theta}_i]$, then $G(\theta) = F(\bar{\theta}_i)$ and $G^-(\theta) = F(\underline{\theta}_i)$.

Finally, I show that s is feasible. By Proposition 1 (Choquet's Theorem) in KMS, there exists a probability measure λ_s supported on the extreme points of $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, denoted by \mathcal{E} , such that $s = \mathbf{E}[\tilde{s} \mid \tilde{s} \sim \lambda_s]$ for all $\theta \in [\theta_0, \bar{\theta}]$. Since $\omega \sim \text{Unif}[0, 1]$, there exists a measurable function $\tilde{S}: [0, 1] \rightarrow \mathcal{E}$ such that $\tilde{S}(\omega) \sim \lambda_s$. In other words, the random variable ω draws an extreme point $\tilde{S}(\omega)$ of $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$ according to λ_s .

For each extreme point \tilde{s} , let $\chi_{\tilde{s}}$ be a deterministic allocation that induces \tilde{s} . Define the stochastic allocation $\chi(\theta, \omega) = \chi_{\tilde{S}(\omega)}(\theta)$. Conditional on the realization of ω , the allocation $\chi(\cdot, \omega)$ induces $S(\omega)$. Therefore, for every $\theta \in [\theta_0, \bar{\theta}]$,

$$\mathbf{E}_\omega[S(\chi(\theta, \omega), G_{\chi(\cdot, \omega)})] = \mathbf{E}_\omega[\tilde{S}(\omega)(\theta)] = \int_{\mathcal{E}} \tilde{s}(\theta) d\lambda_s(\tilde{s}) = s(\theta).$$

For $\theta < \theta_0$, we have $s(\theta) = 0$ is induced by $\chi(\theta, \omega) = 0$ because $S(0, G_\chi) = 0$. Thus, s is feasible. \square

B.2 Proofs of Section 3

B.2.1 Proofs of Section 3.1

Proof of Proposition 2. Because it is optimal for the seller to set $U(\theta_0) = 0$ and $p(\theta_0) = \theta_0 s(\theta_0) + v(\theta_0)$, revenue reduces to

$$R(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \left(J(\theta) s(\theta) + v(\theta) - \frac{1 - F(\theta)}{f(\theta)} v'(\theta) \right) dF(\theta). \quad (\text{B.1})$$

The following lemma follows from applying Theorem 4 (Fan-Lorentz) in KMS to $R(\theta_0)$

Lemma B.1. *Assume $J(\theta)$ is increasing. For any two incentive-compatible mechanisms $(s_1(\theta), p_1(\theta))$ and $(s_2(\theta), p_2(\theta))$ with the same cutoff θ_0 and $U(\theta_0) = 0$, if $s_2 \in \text{MPS}(s_1)$ (i.e., $s_1 \succ s_2$ in the majorization order), then $(s_1(\theta), p_1(\theta))$ generates higher revenue than $(s_2(\theta), p_2(\theta))$.*

As the seller offers more levels of positional goods, it is feasible to offer a finer allocation (in terms of majorization) with the same exclusion level θ_0 , which by Lemma B.1, increases revenue.

Thus, the revenue-maximizing mechanism fully separates the participants (i.e., $s(\theta) = F(\theta) \cdot \mathbf{1}_{\theta \geq \theta_0^*}$) and the optimal cutoff type θ_0^* is given by

$$\begin{aligned} \theta_0^* &= \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} \left(J(\theta) F(\theta) + v(\theta) - \frac{1 - F(\theta)}{f(\theta)} v'(\theta) \right) dF(\theta) \\ &= \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) F(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \end{aligned}$$

If $J(\theta)$ is not monotonic, define $\tilde{J}(\theta; \theta_0) = \int_{\theta_0}^{\theta} J(t) dF(t)$ and apply the ironing technique (Toikka, 2011; Kleiner, Moldovanu, and Strack, 2021). Let

$$K(\tau) = \text{conv } \tilde{J}(F^{-1}(\tau))$$

denote its convex hull on $[F(\theta_0), 1]$ in quantile space $\tau = F(\theta)$. Then, the revenue-maximizing mechanism excludes types below θ_0^* and pools types into the same positional good level if K is affine and separates types if $K \circ F = \tilde{J}$. The optimal cutoff type is given by $\theta_0^* = \arg \max_{\theta_0} \int_{\theta_0}^{\bar{\theta}} F(\theta) dK(F(\theta)) + v(\theta_0)(1 - F(\theta_0))$. \square

Proof of Corollary 2.1. The following lemma shows that $J_v(\theta)$ is increasing if $J(\theta)$ is increasing under the assumptions on $v(\theta)$.

Lemma B.2. *If $J(\theta)$ is increasing, $v'(\theta) > 0$, and $v''(\theta) \leq 0$, then $J_v(\theta) = v(\theta) - \frac{1-F(\theta)}{f(\theta)}v'(\theta)$ is increasing.*

Proof. Define the inverse hazard rate of F as $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$. Thus,

$$\begin{aligned} J'_v(\theta) &= v'(\theta) - I'(\theta)v'(\theta) - h(\theta)v''(\theta) \\ &= J'(\theta)v'(\theta) - h(\theta)v''(\theta) \geq 0 \end{aligned}$$

because $J'(\theta) \geq 0$, $v'(\theta) > 0$, $h(\theta) > 0$, and $v''(\theta) \leq 0$. Hence, $J_v(\theta)$ is increasing. \square

Define the marginal revenue function as $\psi(\theta) = J(\theta)F(\theta) + J_v(\theta)$. Let $\theta_1 = \inf\{\theta \in [0, \bar{\theta}]: J(\theta) > 0\}$ denote the optimal cutoff type in the absence of intrinsic value (and hence $J_v(\theta) \equiv 0$), which satisfies $J(\theta_1) = 0$. We have

$$\psi(\theta_1) = J(\theta_1)F(\theta_1) + J_v(\theta_1) = J_v(\theta_1) = v(\theta_1) - \theta_1 v'(\theta_1) \geq 0 \quad (\text{B.2})$$

by the concavity of $v(\theta)$ and $v(0) = 0$. Moreover, because $J(\theta)$ and $J_v(\theta)$ are increasing, we have

$$\psi'(\theta) = J'(\theta)F(\theta) + J(\theta)f(\theta) + J'_v(\theta) > 0 \text{ for all } \theta > \theta_1. \quad (\text{B.3})$$

Thus, we have $\psi(\theta) > 0$ and thus $R'(\theta) < 0$ for all $\theta > \theta_1$. Therefore, the optimal cutoff satisfies $\theta_0^* \leq \theta_1$.

In particular, if $v(\theta) = \alpha\theta$ for some $\alpha \geq 0$, then $J_v(\theta) = \alpha J(\theta)$, so $\psi(\theta) = (F(\theta) + \alpha)J(\theta)$, which is positive if and only if $J(\theta)$ is positive. Thus, $\theta_0^* = \theta_1$.

(ii) Because $J(\theta)$ is increasing, $J(\theta)F(\theta) > J(0)F(1) = -1/f(0)$. Thus, if $J_v(0) - 1/f(0) = v(0) - (v'(0) + 1)/f(0) \geq 0$, the integrand is always positive, so no exclusion is optimal (i.e., $\theta_0^* = 0$). \square

Proof of Proposition 3. Denote by p the price of the positional good. The cutoff type $\theta(p)$ indifferent between buying and not buying is given by

$$\theta(p) \frac{1 + F(\theta(p))}{2} - p + v(\theta(p)) = 0 \implies p = \theta(p) \frac{1 + F(\theta(p))}{2} + v(\theta(p)).$$

Because $v'(\theta) \geq 0$, all types above $\theta(p)$ will buy the good. Thus, by selling a single good, the seller's maximum revenue is

$$R_1 = \max_{\theta} \left(\theta \frac{1 + F(\theta)}{2} + v(\theta) \right) (1 - F(\theta)) \geq \frac{1}{2} \max_{\theta} (\theta + v(\theta)) (1 - F(\theta)) \quad (\text{B.4})$$

because $\frac{1+F(\theta)}{2}\theta \geq \frac{1}{2}\theta$. Assume first for simplicity $v(\theta) = 0$, so we have $R_1 \geq \frac{1}{2} \max_{\theta} \theta(1 - F(\theta))$.

Consider the auxiliary problem of selling an indivisible good to one buyer, in which a standard extreme-point argument implies a posted-price mechanism is optimal (see, e.g., [Börgers, 2015](#), Proposition 2.5). Formally, let q denote the probability that the agent receives the good, and the set of incentive-compatible allocation is given by $\mathcal{M} = \{q: [0, \bar{\theta}] \rightarrow [0, 1] \mid q \text{ increasing}\}$. The agent's payoff is $u(p, q, \theta) = \theta q - p$. Then, the standard argument implies that

$$\max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) = \max_p p(1 - F(p)),$$

and the optimal mechanism is q^* is an extreme point of \mathcal{M} —i.e., a posted-price mechanism $q^*(\theta) = \mathbf{1}[\theta \geq p^*]$, where $p^* \in \arg \max_p p(1 - F(p))$.

Let R^* denote the maximum revenue from selling positional goods. Because $\text{MPS}_0(F) \subseteq \mathcal{M}$, we have

$$R^* = \max_{s \in \text{MPS}_0(F)} \int_0^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) < \max_{q \in \mathcal{M}} \int_0^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) = \max_p p(1 - F(p)) \leq 2R_1.$$

The inequality is strict because any maximizer \tilde{q}^* of the auxiliary problem satisfies $\tilde{q}^*(\theta) = 1$ on $[\tilde{p}^*, \bar{\theta}]$ for some $\tilde{p}^* < \bar{\theta}$, and thus $\tilde{q}^* \notin \text{MPS}_0(F)$.

Now consider the general case where $v(\theta) \geq 0$ and $v'(\theta) \geq 0$. I show that the same bound continues to hold. In the auxiliary problem, the agent's payoff becomes $u(q, p, \theta) = (\theta + v(\theta))q - p$. We have

$$\begin{aligned} R^* &= \max_{s \in \text{MPS}_0(F), \theta_0} \left[\int_0^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) \right] \\ &< \max_{q \in \mathcal{M}, \theta_0} \left[\int_{\theta_0}^{\bar{\theta}} J(\theta)q(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)) \right] \\ &\leq \max_{\theta_0} \left[\max_{\theta \geq \theta_0} \theta(1 - F(\theta)) + v(\theta_0)(1 - F(\theta_0)) \right]. \end{aligned}$$

In order to establish that $R^* < 2R_1$, it remains to show the following claim:

Claim. Fix any θ_0 , we have

$$2R_1 \geq \theta(1 - F(\theta)) + v(\theta_0)(1 - F(\theta_0)) \text{ for all } \theta \geq \theta_0.$$

Proof of the claim. For all $\theta \geq \theta_0$, because $v(\theta)$ is increasing, we have $v(\theta) \geq v(\theta_0)$ and thus

$$R_1 \geq \left(\theta \frac{1+F(\theta)}{2} + v(\theta) \right) (1-F(\theta)) \geq \left(\theta \frac{1+F(\theta)}{2} + v(\theta_0) \right) (1-F(\theta)).$$

Also,

$$R_1 \geq \left(\theta_0 \frac{1+F(\theta_0)}{2} + v(\theta_0) \right) (1-F(\theta_0)) \geq v(\theta_0)(1-F(\theta_0)).$$

There are two cases. First, if $v(\theta_0)(1-F(\theta_0)) \geq \theta(1-F(\theta))$, then the second inequality above implies

$$2R_1 \geq 2v(\theta_0)(1-F(\theta_0)) \geq \theta(1-F(\theta)) + v(\theta_0)(1-F(\theta_0)).$$

Otherwise, if $v(\theta_0)(1-F(\theta_0)) < \theta(1-F(\theta))$, then

$$0 \leq v(\theta_0) < \frac{\theta(1-F(\theta))}{1-F(\theta_0)}.$$

It is enough to show that

$$\left(\theta \frac{1+F(\theta)}{2} + v(\theta_0) \right) (1-F(\theta)) \geq \frac{1}{2} [\theta(1-F(\theta)) + v(\theta_0)(1-F(\theta_0))],$$

or equivalently,

$$\frac{1}{2} (\theta F(\theta)(1-F(\theta)) + v(\theta_0) [2(1-F(\theta)) - (1-F(\theta_0))]) \geq 0.$$

This expression is affine in $v(\theta_0)$. Therefore, it is enough to check the two endpoints $v(\theta_0) = 0$ and $v(\theta_0) = \frac{\theta(1-F(\theta))}{1-F(\theta_0)}$. At $v(\theta_0) = 0$, the expression equals

$$\frac{1}{2} \theta F(\theta)(1-F(\theta)) \geq 0.$$

At $v(\theta_0) = \frac{\theta(1-F(\theta))}{1-F(\theta_0)}$, the expression equals

$$\frac{1}{2} \left[\theta F(\theta)(1-F(\theta)) + \frac{\theta(1-F(\theta))}{1-F(\theta_0)} [2(1-F(\theta)) - (1-F(\theta_0))] \right] = \frac{1}{2} \theta (1-F(\theta))^2 \frac{1+F(\theta_0)}{1-F(\theta_0)} \geq 0.$$

Thus,

$$\left(\theta \frac{1 + F(\theta)}{2} + v(\theta_0) \right) (1 - F(\theta)) \geq \frac{1}{2} [\theta(1 - F(\theta)) + v(\theta_0)(1 - F(\theta_0))].$$

Since R_1 is at least the left-hand side, we have

$$2R_1 \geq \theta(1 - F(\theta)) + v(\theta_0)(1 - F(\theta_0)).$$

Therefore, the desired inequality holds for every $\theta \geq \theta_0$. □

Hence, we have established that $R_1 > \frac{1}{2}R^*$. □

B.2.2 Proofs of Section 3.3

Proof of Proposition 4. (i) First, I show the following lemma. The lemma follows from applying Fan-Lorentz theorem to

$$W(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} \right) (s(\theta) + v'(\theta)) + U(\theta_0) dF(\theta) \quad (\text{B.5})$$

and noting that $\frac{1-F(\theta)}{f(\theta)}$ is decreasing (increasing) under IFR (DFR).

Lemma B.3. *For any two incentive-compatible mechanisms $(s_1(\theta), p_1(\theta))$ and $(s_2(\theta), p_2(\theta))$ with the same cutoff θ_0 and $U(\theta_0) = 0$, if $s_2 \in \text{MPS}(s_1)$ (i.e., $s_1 \succ s_2$ in the majorization order), then $(s_1(\theta), p_1(\theta))$ generates lower consumer surplus than $(s_2(\theta), p_2(\theta))$.*

The lemma implies that, fixing the exclusion level θ_0 , the consumer surplus decreases (increases) when the status profile becomes finer under IFR (DFR).

In addition, denote by s_k^* the revenue-maximizing status profile chosen by the seller when she offers $k \geq 1$ levels of positional goods. Under Myerson's regularity (which is implied by IFR), by Lemma B.1, $s_k^* \in \text{MPS}(s_{k+1}^*)$. Thus, by Lemma B.3, under IFR (DFR), the consumer surplus is decreasing (increasing) in k , the number of positional good levels the seller offers.

(ii) When the number of levels is unconstrained, because $J(\theta)$ is increasing, the seller chooses $s^*(\theta) = F(\theta)$ for all $\theta \in [\theta_0, \bar{\theta}]$. Thus, consumer surplus is given by

$$W(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \frac{1 - F(\theta)}{f(\theta)} (F(\theta) + v'(\theta)) dF(\theta),$$

which is increasing in θ_0 .

When the number of levels is constrained to a single level, I show a stronger result: holding the status profile of consumers of higher tiers than the basic tier fixed, decreasing the exclusion level θ_0 increases consumer surplus under IFR. The case where the number of levels is constrained to a single tier because there are no higher tiers than the basic tier.

Fix an allocation $s(\theta)$ and its exclusion level θ_0 . If $s(\theta)$ is *strictly* increasing at θ_0 , then decreasing θ_0 to $\theta_0 - \varepsilon$ increases the consumer surplus for all $\theta \in [\theta_0 - \varepsilon, \theta_0)$ and does not affect the status for all $\theta \in [\theta_0, \bar{\theta}]$. Hence, the consumer surplus increases.

If $s(\theta)$ is constant in a neighborhood of θ_0 , then there exists some $\theta_1 \in (\theta_0, \bar{\theta}]$ such that $s(\theta) = s_0$ is constant on $[\theta_0, \theta_1]$ and $s(\theta) > s_0$ for $\theta > \theta_1$. Thus, consumer surplus is

$$W(\theta_0) = \int_{\theta_0}^{\theta_1} \frac{1 - F(\theta)}{f(\theta)} (v'(\theta) + F(\theta_1)/2 + F(\theta_0)/2) dF(\theta) + \int_{\theta_1}^{\bar{\theta}} \frac{1 - F(\theta)}{f(\theta)} (v'(\theta) + s(\theta)) dF(\theta),$$

with derivative given by

$$W'(\theta_0) = -(1 - F(\theta_0)) \left(v'(\theta_0) + \frac{F(\theta_1) + F(\theta_0)}{2} \right) + \frac{f(\theta_0)}{2} \int_{\theta_0}^{\theta_1} (1 - F(\theta)) d\theta \quad (\text{B.6})$$

Finally, to prove that $W'(\theta_0) \leq 0$ under IFR, I show that $W'(\theta_0) \leq 0$ holds under a weaker condition.

Lemma B.4. $W'(\theta_0) \leq 0$ if

$$\frac{1}{F(\theta_1) - F(\theta_0)} \int_{\theta_0}^{\theta_1} \frac{1 - F(t)}{f(t)} dF(t) \leq \frac{1 - F(\theta_0)}{f(\theta_0)}. \quad (\text{B.7})$$

The condition holds under IFR.

Proof. Rearranging the inequality yields

$$\frac{f(\theta_0)}{2} \int_{\theta_0}^{\theta_1} (1 - F(t)) dt \leq (1 - F(\theta_0)) \frac{F(\theta_1) - F(\theta_0)}{2}.$$

Substituting this into (B.6) yields

$$\begin{aligned} W'(\theta_0) &\leq -(1 - F(\theta_0)) \left[v'(\theta_0) + \frac{F(\theta_0) + F(\theta_1)}{2} \right] + \frac{1 - F(\theta_0)}{2} [F(\theta_1) - F(\theta_0)] \\ &= -(1 - F(\theta_0)) [v'(\theta_0) + F(\theta_0)] \leq 0. \end{aligned}$$

The last inequality is strict for all $\theta \in (0, \bar{\theta})$.

If F satisfies IFR, then $\frac{1-F(t)}{f(t)}$ is decreasing, so condition B.7 holds. \square

By the lemma above, IFR implies $W'(\theta_0) \leq 0$. Hence, decreasing θ_0 increases the consumer surplus $W(\theta_0)$ in both cases.

Finally, the following example shows that when IFR fails, reducing exclusion may lower consumer surplus.

Example (Weibull distribution). Assume $v(\theta) = 0$ and $F(\theta) = 1 - e^{-\sqrt{\theta}}$ on $[0, \infty)$, which satisfies DFR. Consider the case where the seller offers a single tier of positional good, so that $s(\theta) = (1 + F(\theta_0))/2$. Then, the seller's optimal exclusion cutoff is $\theta_0^* \approx 4.56$. Because

$$W'(\theta_0) = \frac{f(\theta_0)}{2} \int_{\theta_0}^{\infty} (1 - F(\theta)) d\theta - \frac{1 + F(\theta_0)}{2} (1 - F(\theta_0))$$

We have $W'(\theta_0^*) \approx -0.075$. Thus, when the seller offers a single tier of positional good, expanding participation reduces consumer surplus. □

Proof of Proposition 5. By the argument in the main text, since $U(\theta_0) = v(0) \cdot \mathbf{1}_{[\theta_0=0]}$, the consumer surplus can be written as

$$W(\theta_0) = \int_{\theta_0}^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} (s(\theta) + v'(\theta)) + v(0) \cdot \mathbf{1}_{[\theta_0=0]} \right) dF(\theta). \quad (\text{B.8})$$

First, I show that it is optimal to set $\theta_0 = 0$. Fix an exclusion level θ_0 and let $s^\dagger(\theta)$ denote the maximizer of $W(\theta_0)$ given θ_0 . If $s^\dagger(\theta)$ is *strictly* increasing at θ_0 , then decreasing θ_0 to $\theta_0 - \varepsilon$ increases the consumer surplus for all $\theta \in [\theta_0 - \varepsilon, \theta_0)$ and does not affect the consumer surplus for all $\theta \in [\theta_0, \bar{\theta}]$, so the consumer surplus increases.

If $s^\dagger(\theta)$ is constant in a neighborhood of θ_0 , then there exists some $\theta_1 \in (\theta_0, \bar{\theta}]$ such that $s^\dagger(\theta) = s_0$ is constant on $[\theta_0, \theta_1]$ and $s^\dagger(\theta) > s_0$ for $\theta > \theta_1$. Because $s^\dagger(\theta)$ maximizes $W(\theta_0)$, by Proposition 2 in KMS, we have

$$\frac{1}{F(\theta) - F(\theta_0)} \int_{\theta_0}^{\theta} \frac{1 - F(t)}{f(t)} dF(t) \geq \frac{1}{F(\theta_1) - F(\theta_0)} \int_{\theta_0}^{\theta_1} \frac{1 - F(t)}{f(t)} dF(t) \text{ for all } \theta \in [\theta_0, \theta_1].$$

The inequality holds at the limit $\theta \rightarrow \theta_0^+$, so

$$\frac{1 - F(\theta_0)}{f(\theta_0)} \geq \frac{1}{F(\theta_1) - F(\theta_0)} \int_{\theta_0}^{\theta_1} \frac{1 - F(t)}{f(t)} dF(t)$$

Then, by Lemma B.4, we have $W'(\theta_0) \leq 0$. Hence, it is optimal to set $\theta_0 = 0$ in both cases.

Therefore, the optimization problem reduces to maximizing

$$\max_{s \in \text{MPS}(F)} \int_0^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} (s(\theta) + v'(\theta)) + v(0) \right) dF(\theta).$$

Let $H(\theta) = \int_0^\theta \frac{1-F(t)}{f(t)} dF(t)$ and $\text{conv } H$ denote its convex hull in quantile space. By Proposition 2 in KMS, we have

1. If $\frac{1-F(\theta)}{f(\theta)}$ is decreasing (i.e., IFR), then $s^*(\theta) = 1/2$.
2. If $\frac{1-F(\theta)}{f(\theta)}$ is increasing (i.e., DFR), then $s^*(\theta) = F(\theta)$.
3. If $\frac{1-F(\theta)}{f(\theta)}$ is not monotonic, then s^* separates types when $\text{conv } H = H$ and pools them into the same level when $\text{conv } H$ is affine.

When the failure rate is single-peaked (single-dipped), $H = \int_0^\theta \frac{1-F(t)}{f(t)} dF(t)$ is concave-convex (convex-concave), and the results in (iii) and (iv) follow immediately from 3. \square

B.3 Proofs of Section 5

Lemma B.5 (Deterministic monotone implementation). *If $J(\theta)$ is increasing, the optimal solution is induced by a deterministic increasing allocation $\chi : \Theta \rightarrow \tilde{X}$ such that*

$$x(\theta) = \chi(\theta), \quad s(\theta) = S(\chi(\theta), G_\chi).$$

Proof of Lemma B.5. Fix any feasible randomized allocation $\chi(\theta, \omega)$. For each realization ω , let $\chi^\uparrow(\cdot, \omega)$ be the increasing rearrangement of $\chi(\cdot, \omega)$ with respect to F . Then, $G_{\chi^\uparrow}(\cdot | \omega) = G_\chi(\cdot | \omega)$. Since $S(x, G)$ is increasing in x for fixed G , the function $x + S(x, G_\chi(\cdot | \omega))$ is increasing in x . Hence, because J is increasing, the monotone rearrangement inequality gives

$$\begin{aligned} & \int_{\Theta} J(\theta) [\chi^\uparrow(\theta, \omega) + S(\chi^\uparrow(\theta, \omega), G_{\chi^\uparrow}(\cdot | \omega))] dF(\theta) \\ & \geq \int_{\Theta} J(\theta) [\chi(\theta, \omega) + S(\chi(\theta, \omega), G_\chi(\cdot | \omega))] dF(\theta). \end{aligned}$$

Moreover, since $\chi^\uparrow(\cdot, \omega)$ and $\chi(\cdot, \omega)$ have the same distribution under F ,

$$\int_{\Theta} c(\chi^\uparrow(\theta, \omega)) dF(\theta) = \int_{\Theta} c(\chi(\theta, \omega)) dF(\theta).$$

Thus we can restrict attention to randomized allocations that are increasing in θ in every realization ω .

Now suppose $\chi(\cdot, \omega)$ is increasing for every ω . Define the deterministic average allocation $\bar{x}(\theta) = \mathbf{E}_\omega[\chi(\theta, \omega)]$, which is also increasing. The quality part of revenue is unchanged because

$$\mathbf{E}_\omega \int_{\Theta} J(\theta) \chi(\theta, \omega) dF(\theta) = \int_{\Theta} J(\theta) \bar{x}(\theta) dF(\theta).$$

By Jensen's inequality and the strict convexity of c ,

$$\mathbf{E}_\omega \int_{\Theta} c(\chi(\theta, \omega)) dF(\theta) \geq \int_{\Theta} c(\bar{x}(\theta)) dF(\theta).$$

The inequality is strict unless $\chi(\theta, \omega)$ is constant in ω for almost every θ .

It remains to compare the status part. Let

$$s_\omega(\theta) = S(\chi(\theta, \omega), G_{\chi(\cdot | \omega)}), \quad \bar{s}(\theta) = S(\bar{x}(\theta), G_{\bar{x}}).$$

Since $\bar{x}(\theta) = \mathbf{E}_\omega[\chi(\theta, \omega)]$ and each $\chi(\cdot, \omega)$ is increasing in θ , whenever \bar{x} is flat on an interval, each $\chi(\cdot, \omega)$ must also be flat on that interval for almost every ω . Hence, every pooling interval of \bar{x} is contained in a pooling interval of $\chi(\cdot, \omega)$. Therefore, $\bar{s} \succ s_\omega$ in the majorization order. By Lemma B.1, because J is increasing, this implies

$$\int_{\Theta} J(\theta) \bar{s}(\theta) dF(\theta) \geq \int_{\Theta} J(\theta) s_\omega(\theta) dF(\theta)$$

for almost every ω , and therefore

$$\int_{\Theta} J(\theta) \bar{s}(\theta) dF(\theta) \geq \mathbf{E}_\omega \int_{\Theta} J(\theta) s_\omega(\theta) dF(\theta)$$

in expectation.

Combining the three parts, the deterministic increasing allocation \bar{x} (strictly) dominates the original (nondegenerate) randomized allocation. Finally, it is straightforward to verify that the deterministic allocation \bar{x} induces the interim quality \bar{x} and status $\bar{s} = S(\bar{x}(\theta), G_{\bar{x}})$ at the same time. \square

Proof of Proposition 8. By Lemma B.5, it is without loss of optimality to restrict attention to deterministic increasing allocation rules $\chi : \Theta \rightarrow \tilde{X}$ such that $x(\theta) = \chi(\theta)$ and $s(\theta) = S(\chi(\theta), G_\chi)$.

Consider the relaxed optimization problem that ignores the compatibility constraint:

$$\max_{s \in \text{MPS}_w(F), x \text{ increasing}} \int_0^{\bar{\theta}} [J(\theta) s(\theta) + J(\theta) x(\theta) - c(x(\theta))] dF(\theta).$$

Let $\theta_0^* = \inf\{\theta \in [0, \bar{\theta}]: J(\theta) > 0\}$ denote the cutoff type such that $J(\theta) \leq 0$ for $\theta < \theta_0^*$ and $J(\theta) \geq 0$ for $\theta \geq \theta_0^*$.

Consider first the quality $x(\theta)$. Using pointwise maximization, the solution is

$$x^*(\theta) = \begin{cases} 0, & \text{if } \theta < \theta_0^*, \\ c'^{-1}(J(\theta)), & \text{if } \theta \geq \theta_0^*. \end{cases}$$

When J is increasing, $x^*(\theta)$ is increasing. For all $\theta \geq \theta_0^*$, $x^*(\theta)$ is strictly increasing if $J(\theta)$ is strictly increasing, and $x^*(\theta)$ is constant if $J(\theta)$ is constant.

Now consider the status $s(\theta)$. By the same argument as in Proposition 2, the revenue-maximizing status allocation excludes types below θ_0^* . On $[\theta_0^*, \bar{\theta}]$, on intervals where $J(\theta)$ is strictly increasing, the optimal status is $s^*(\theta) = F(\theta)$. On any maximal interval $[\underline{\theta}_i, \bar{\theta}_i] \subseteq [\theta_0^*, \bar{\theta}]$ on which $J(\theta) = \bar{J}_i$ is constant, we have

$$\int_{\underline{\theta}_i}^{\bar{\theta}_i} J(\theta) s(\theta) dF(\theta) = \bar{J}_i \int_{\underline{\theta}_i}^{\bar{\theta}_i} s(\theta) dF(\theta),$$

Thus, it is without loss of optimality to set

$$s^*(\theta) = \frac{F(\underline{\theta}_i) + F(\bar{\theta}_i)}{2}, \quad \text{for all } \theta \in [\underline{\theta}_i, \bar{\theta}_i].$$

On the participating interval $[\theta_0^*, \bar{\theta}]$, in the regions where J is strictly increasing, the quality scheme $x^*(\theta)$ is strictly increasing and induces separating status $s^*(\theta) = F(\theta)$; in each maximal interval $[\underline{\theta}_i, \bar{\theta}_i] \subseteq [\theta_0^*, \bar{\theta}]$ on which $J(\theta)$ is constant, the quality scheme $x^*(\theta)$ is constant and induces constant status $s^*(\theta) = (F(\underline{\theta}_i) + F(\bar{\theta}_i))/2$. Hence, in either case, we have

$$s^*(\theta) = S(x^*(\theta), G_{x^*}) \quad \text{for all } \theta \geq \theta_0^*,$$

i.e., the optimal status and quality schemes are induced by the same allocation $\chi^* = x^*$. Moreover, the exclusion cutoff θ_0^* is the same for both $s^*(\theta)$ and $x^*(\theta)$. Hence, the solution to the relaxed problem is compatible and solves the original problem. \square

Proof of Proposition 7. I prove the result for revenue maximization (i.e., $z = J$) only; others cases ($z = J_\lambda$ and $z = \frac{1-F(\theta)}{f(\theta)}$) can be proven analogously.

The revenue maximization problem can be written as

$$[\text{M}] \quad \sup_{\tilde{s} \in \text{conv } \mathcal{F}, \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta) \tilde{s}(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \quad (\text{B.9})$$

For any fixed θ_0 , the objective is linear in \tilde{s} . Thus, the supremum over a set \mathcal{F} is the same as the supremum over its convex hull. Since each element of \mathcal{F} is of the form $\tilde{s} = \phi \circ s$ for some $s \in \text{ext MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, the problem can be rewritten as

$$[\text{M}] \quad \sup_{s \in \text{ext MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})} \int_{\theta_0}^{\bar{\theta}} J(\theta) \phi(s(\theta)) \, dF(\theta) + v(\theta_0)(1 - F(\theta_0)). \quad (\text{B.10})$$

Now consider a *relaxed* problem [R] where the constraint is relaxed to $s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$.

$$[\text{R}] \quad \max_{s \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})} \int_{\theta_0}^{\bar{\theta}} J(\theta) \phi(s(\theta)) \, dF(\theta) + v(\theta_0)(1 - F(\theta_0)).$$

The rest of the proof solves the relaxed problem and then show that its solution is an extreme point of $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, so it is also the solution to the original problem.

(i) Suppose ϕ is strictly increasing and convex. Let $\hat{\theta}_0 = J^{-1}(0) \equiv \inf\{\theta \in [0, \bar{\theta}]: J(\theta) > 0\}$. Then, for all $\theta \geq \hat{\theta}_0$, the kernel $K(s, \theta) = J(\theta)\phi(s)$ is convex in s and supermodular in (s, θ) , since $J(\theta)$ is positive and increasing. By the Fan–Lorentz theorem, a finer status profile weakly increases expected revenue. Hence, if the cutoff type is $\hat{\theta}_0$, the revenue-maximizing status profile is full separation $s^*(\theta) = F(\theta)$.

Then, I show that the optimal cutoff type is indeed $\hat{\theta}_0$. Because $v(\theta) = \alpha\theta$, we have $J_v(\theta) = \alpha J(\theta)$, so the objective can be written as

$$\int_{\theta_0}^{\bar{\theta}} J(\theta) \phi(s(\theta)) + J_v(\theta) \, dF(\theta) = \int_{\theta_0}^{\bar{\theta}} J(\theta) (\phi(s(\theta)) + \alpha) \, dF(\theta),$$

where the integrand is positive if and only if $\theta > \hat{\theta}_0$. Thus, the optimal cutoff type is $\hat{\theta}_0$. Therefore, the solution to the relaxed problem [R] is $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \hat{\theta}_0]$. Because $s^*(\theta)$ is also an extreme point of $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, it is also the solution to the original problem [M].

(ii) Suppose ϕ is strictly increasing and concave. Then, I use the optimal control method to show that $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$ for some θ_0^* if $J(\theta)\phi'(F(\theta))$ is increasing in θ .

Define $D(\theta) = \int_{\theta}^{\bar{\theta}} (F(t) - s(t)) \, dF(t) \geq 0$. Ignoring the monotonicity constraint on s , the rewrite the feasibility constraint as $D(\theta) \geq 0$ and $D'(\theta) = (s(\theta) - F(\theta))f(\theta)$. Thus, the Hamiltonian is

$$H = J(\theta)\phi(s)f(\theta) + \lambda D + \Lambda(s - F)f, \quad (\text{B.11})$$

where D is the state variable, s is the control variable, $\lambda(\theta) \geq 0$ is the Lagrangian multiplier on $D(\theta) \geq 0$, and $\Lambda(\theta)$ is the costate variable associated with \dot{D} . The Pontryagin's

maximum principle implies that

$$\frac{\partial H}{\partial s} = J(\theta)\phi'(s(\theta))f(\theta) + \Lambda(\theta)f(\theta) = 0, \quad (\text{B.12})$$

$$\frac{\partial H}{\partial D} = \lambda(\theta) = -\dot{\Lambda}(\theta), \quad (\text{B.13})$$

$$\lambda(\theta)D(\theta) = 0, \quad \lambda(\theta) \geq 0, \quad (\text{B.14})$$

$$J(\theta_0)\phi(s(\theta_0))f(\theta_0) - v'(\theta_0)(1 - F(\theta_0)) + v(\theta_0)f(\theta_0) = 0, \quad (\text{B.15})$$

$$\Lambda(\theta_0), \Lambda(\bar{\theta}) \text{ free.} \quad (\text{B.16})$$

Because the Hamiltonian H is concave in s and D , the necessary conditions are also sufficient. Suppose $-\Lambda(\theta) = J(\theta)\phi'(F(\theta))$ is increasing in θ . Then, the solution to the relaxed version of [R] (which ignores the monotonicity constraint) is $s^*(\theta) = F(\theta) \cdot \mathbf{1}[\theta \geq \theta_0^*]$ for some θ_0^* , which satisfies the monotonicity constraint and thus is the solution to [R]. Moreover, because $s^*(\theta)$ is an extreme point of $\text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$, it is also the solution to the original problem [M]. \square

Proof of Proposition 9. Let $\text{MPS}(\mathbf{1}_{[\theta_0, \bar{\theta}]}) - (F)$ denote the set of increasing functions $s: \Theta \rightarrow (-\infty, 1]$ that satisfies $s \leq \hat{s}$ for some $\hat{s}: \Theta \rightarrow [0, 1]$ such that $\hat{s} \in \text{MPS}(F \cdot \mathbf{1}_{[\theta_0, \bar{\theta}]})$.

By the same standard arguments, it is optimal to set $U(\theta_0) = 0$, and the revenue maximization problem is

$$\max_{s \in \text{MPS}(\mathbf{1}_{[\theta_0, \bar{\theta}]}) - (F), \theta_0} \int_{\theta_0}^{\bar{\theta}} J(\theta)s(\theta) dF(\theta) + v(\theta_0)(1 - F(\theta_0)).$$

Let $\hat{\theta}_0^*$ denote the cutoff such that $J(\theta) \geq 0$ if and only if $\theta \geq \hat{\theta}_0^*$. For all $\theta \geq \hat{\theta}_0^*$, because $J(\theta) \geq 0$ is increasing, we have $s^*(\theta) = F(\theta)$.

For all $\theta < \hat{\theta}_0^*$, we have $J(\theta) < 0$, so the revenue is decreasing in $s(\theta)$. Because $U(\theta_0) = 0$, to maintain $U(\theta) \geq 0$ (IR), the allocation must satisfy $U'(\theta) = s(\theta) + v'(\theta) \geq 0$. Consequently, the lowest permissible allocation is $s^*(\theta) = -v'(\theta)$. Moreover, $p(\theta) = -\theta v'(\theta) + v(\theta) \geq 0$ for all $\theta \leq \hat{\theta}_0^*$ follows from the concavity of $v(\theta)$ and that $v(0) \geq 0$.

Compared to the revenue-maximizing mechanism characterized in Proposition 2, delaying service increases revenue because that mechanism is feasible. Moreover, delaying service decreases consumer surplus because it effectively changes exclusion from θ_0^* to $\hat{\theta}_0^*$ (below which types have zero payoffs), which, by Corollary 2.1, is higher than the optimal exclusion in Proposition 2 (i.e., $\hat{\theta}_0^* \geq \theta_0^*$). \square

Proof of Proposition 10. (i) Let $L(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$. After exchanging the order of integration

and setting $U(\theta_0) = 0$, the revenue can be written as

$$R = \int_0^{\theta_0} \left(L(\theta)s(\theta) + v(\theta) + \frac{F(\theta)}{f(\theta)}v'(\theta) \right) dF(\theta) \quad (\text{B.17})$$

Then, the proof is similar to that of Proposition 2.

(ii) The proof is similar to that of Proposition 3. The only difference is that because $v'(\theta) \leq 0$, all types $\theta \leq \theta(p)$ buy the good. Thus, the auxiliary problem becomes a screening problem in procurement where the *lower* price get the good.

(iii) The proof is similar to that of Corollary 5. The only difference is that the consumer surplus is

$$W = \int_0^{\theta_0} \left(-\frac{F(\theta)}{f(\theta)}(s(\theta) + v'(\theta)) + U(\theta_0) \right) dF(\theta). \quad (\text{B.18})$$

Thus, pooling (separating) all participants $\theta \in [\theta_0, \bar{\theta}]$ is optimal if $\frac{F(\theta)}{f(\theta)}$ is increasing (decreasing).

In the case of pooling, $s(\theta_0) = 1 - F(\theta_0)/2$ for all $\theta \in [0, \theta_0]$.

$$W(\theta_0) = \int_0^{\theta_0} -\frac{F(\theta)}{f(\theta)}(v'(\theta) + 1 - F(\theta_0)/2) dF(\theta) + v(\bar{\theta}) \cdot \mathbf{1}_{[\theta_0=\bar{\theta}]}$$

Because $v'(\theta_0) \leq -1$, we have

$$W'(\theta_0) = -F(\theta_0)(v'(\theta_0) + 1 - F(\theta_0)/2) + \frac{f(\theta_0)}{2} \int_0^{\theta_0} F(\theta) d\theta \geq 0$$

for all $\theta_0 < \bar{\theta}$. Hence, no exclusion is optimal (i.e., $\theta_0^* = \bar{\theta}$).

In the case of separation, we have $s(\theta) = F(\theta) + 1 - F(\theta_0)$ for all $\theta \in [0, \theta_0]$. Thus,

$$W(\theta_0) = \int_0^{\theta_0} -\frac{F(\theta)}{f(\theta)}(v'(\theta) + F(\theta) + 1 - F(\theta_0)) dF(\theta) + v(\bar{\theta}) \cdot \mathbf{1}_{[\theta_0=\bar{\theta}]}$$

and

$$W'(\theta_0) = -F(\theta_0)(v'(\theta_0) + 1) + \int_0^{\theta_0} F(\theta)f(\theta_0) d\theta \geq 0 \text{ for all } \theta_0 < \bar{\theta}$$

because $v'(\theta_0) \leq -1$. Hence, no exclusion is optimal (i.e., $\theta_0^* = \bar{\theta}$). □

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