

Tournaments with Managerial Discretion*

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Abstract

We study tournaments with managerial discretion in hiring. A manager selects a coworker from a pool of candidates and then competes against her in a Lazear–Rosen-style tournament, where the prize is a share of total output. A profit-maximizing principal sets both the prize share and a head start (or handicap)—an advantage (or disadvantage) in output comparison—granted to the manager. The head start affects output through three channels: (i) encouraging the manager, (ii) discouraging the new hire, and (iii) inducing the manager to hire a stronger candidate. The hiring effect dominates the discouragement effect until the strongest candidate is hired; beyond that, any further head start discourages the new hire more than it encourages the manager. Consequently, the optimal contract provides just enough head start to ensure the manager hires the strongest candidate.

Keywords: Tournaments, managerial discretion, sabotage, head start, output-dependent prizes.

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“[W]hen compensation is relative, and when the individuals who do the hiring are to be in the same pool with those hired, there is an incentive to hire people strategically. Incumbents do not want competition from good outsiders, and so they tend to hire lower-quality people than would otherwise be optimal for the firm.”

—Edward P. Lazear, *Personnel Economics*

1 Introduction

In hierarchical organizations, principals often delegate hiring and promotion decisions to managers, who are better positioned to evaluate candidates’ abilities. Under tournament-based incentive schemes, however, the selected candidate becomes both a coworker and a competitor of the incumbent manager. This creates an incentive for the manager to sabotage the selection process by choosing a lower-ability candidate than would otherwise be optimal for the firm in order to prevent competition (Lazear, 1995).¹ For example, in academia, universities face the challenge of ensuring that incumbent faculty members hire the best possible candidates, and tenure is considered essential for this purpose (Carmichael, 1988). In politics, incumbents tend to appoint weaker deputies to prevent future competition. Similar dynamics also arise in corporate environments. As Steve Jobs famously remarked, “A players hire A players, B players hire C players, and C players hire D players.”² In a survey of 336 corporate executives in the U.S. across various industries, Zaman and Lakhani (2024) asked if respondents have “ever observed a colleague disapprove hiring of a high-ability candidate to avoid potential competition for himself or herself.” Among those whose firms operated on relative performance evaluation, over 30% answered in the affirmative.

Despite the observational evidence, tournaments with managerial discretion and sabotage in the hiring process remain underexplored. To fill this gap, we model a two-player tournament in which the manager has discretion over hiring a co-worker, who inherently becomes a competitor. The manager chooses the new hire’s ability and then competes against her in a Lazear-Rosen-style tournament (Lazear and Rosen, 1981), where the winner receives a share of total output. For example, at a technology company, a senior employee is delegated authority to hire a junior colleague for her team. The new hire contributes to team output but also becomes a future rival in an internal tournament for bonuses tied to team performance.

We propose that the principal can offer the manager a *head start* to partially insulate her from

¹Our findings are applicable to both hiring and promotions in tournaments where the firm relies on managerial discretion to select a candidate to be in the same pool with the manager. We use “she” (“he”) to refer to the manager (selected candidate).

²Obtained from Kawasaki (2015). Sullivan (2011) also notes that weak managers often “don’t even try to hire superior talent” because they fear being overshadowed or displaced; instead, they hire employees with lower ability to increase their job security.

potential competition. The head start is an advantage granted to the manager when comparing her output with the new hire's, which can be interpreted as a premium for seniority within the organization (see, e.g., [Konrad \(2002\)](#); [Siegel \(2014\)](#)). For example, in competition for bonuses or promotions, firms can provide incumbency advantages to employees who have been at the firm longer. In organizations, incumbents can receive more credit than they deserve because of seniority. Head starts, which are equivalent to handicapping opponents, are commonly used in tournaments *without* managerial discretion—such as organizational and sports contests—to provide incentives ([Lazear and Rosen \(1981\)](#); [O’Keeffe, Viscusi, and Zeckhauser \(1984\)](#); [Drugov and Ryvkin \(2017\)](#); [Fu and Wu \(2020\)](#)). By contrast, we explore the role of head starts in mitigating managerial sabotage in hiring, in addition to providing incentives in tournaments.

The principal sets the tournament prize as a constant share of total output, and her commitment to this share may stem from equity-based compensation or reputation. In practice, better-performing employees are often rewarded with equity compensation, such as firm stock. In law firms, accounting firms, and investment banks, lower-ranked employees often compete in promotion tournaments for partner positions, which come with compensation tied to firm performance.³ This incentive structure turns the new hire into a collaborator and thus incentivizes the manager to hire a high-ability candidate: although hiring a lower-ability agent still increases her chances of winning, it also reduces the tournament prize. Incentive schemes based on jointly achieved outcomes are common in organizations to motivate agents ([Holmstrom \(1982\)](#); [Dai and Toikka \(2022\)](#); [Dasaratha, Golub, and Shah \(2024\)](#)). In tournaments, output-dependent prizes are used to avoid uncertainty, minimize wage costs, and prevent (peer-to-peer) sabotage and collusion ([Güth et al. \(2016\)](#); [Danilov, Harbring, and Irlenbusch \(2019\)](#); [Glökler, Pull, and Stadler \(2022\)](#)).

We find that the head start has varying effects on the efforts of both the manager and the new hire. First, it biases the tournament in favor of the manager and incentivizes her to exert effort, thereby having an *encouragement effect*. However, its impact on the new hire's effort is rather mixed. On the one hand, the bias discourages the new hire from investing effort, thereby having a *discouragement effect*. On the other hand, it mitigates the competition faced by the manager and leads her to hire a higher-ability agent, which increases the new hire's effort relative to the situation where a lower-ability agent would have been hired. This positive *hiring effect* dominates the discouragement effect in equilibrium until the highest-ability agent is hired. Once the highest-ability candidate is hired, any further head start discourages the new hire more than it encourages the manager, thereby decreasing total output. Therefore, the optimal head start offers just enough head start to induce the

³For example, Goldman Sachs allocated 34% of its net revenue to its compensation pool in 2023 ([The Goldman Sachs Group, Inc., 2023](#)). Partners receive disproportionately large shares relative to lower-ranked employees. These shares may vary across partners and are determined by a compensation committee appointed by the Board of Directors ([Goldman Sachs Group, Inc., 2025](#)). [Baker, Jensen, and Murphy \(1988\)](#) also find that promotions are the primary incentive devices in most organizations.

manager to hire the strongest candidate.

The payout ratio (i.e., the share of total output awarded to the winner) also has mixed effects on the principal's profit. While a higher payout ratio reduces the principal's share of total output, it affects the agents' effort in two ways. On the one hand, a higher payout ratio incentivizes both the manager and the new hire to exert more effort. On the other hand, it affects the manager's choice of the new hire's ability, as the manager faces a trade-off: hiring a higher-ability agent increases the tournament prize but lowers her chances of winning. We characterize the profit-maximizing head start and payout ratio for the principal to take advantage of tournament incentives while mitigating hiring sabotage.⁴

In this paper, we explore the unique dynamics in tournaments that involve managerial discretion in hiring the other player. To the best of our knowledge, we are the first paper in contest theory to study the role of a player's discretion in choosing her competitor, which leads to sabotage during this process. Moreover, we study the use of head starts and output-dependent prizes in tournaments to mitigate hiring sabotage. While the literature has explored the use of head starts (or handicaps) to provide incentives in the tournament stage (e.g., [Lazear and Rosen \(1981\)](#); [O'Keeffe, Viscusi, and Zeckhauser \(1984\)](#); [Drugov and Ryvkin \(2017\)](#)) and tournaments with output-dependent prizes (e.g., [Chung \(1996\)](#); [Baye and Hoppe \(2003\)](#); [Gershkov, Li, and Schweinzer \(2009\)](#); [Güth et al. \(2016\)](#); [Danilov, Harbring, and Irlenbusch \(2019\)](#)), we are the first to explore their roles in mitigating managerial sabotage in the hiring process. Additionally, even in the absence of managerial discretion, we show that giving a head start to the lower-ability agent can decrease total effort in tournaments with output-dependent prizes, contrary to the common wisdom of "leveling the playing field." Our results on the effects of the head start, noise, and abilities on individual efforts are also novel in the setting with output-dependent prizes.

The tournament literature has explored peer-to-peer sabotage (e.g., [Lazear \(1989\)](#); [Skaperdas and Grofman \(1995\)](#); [Chen \(2003\)](#); [Kräkel \(2005\)](#); [Münster \(2007\)](#); [Harbring and Irlenbusch \(2008\)](#); [Gürtler and Münster \(2010\)](#); [Deutscher et al. \(2013\)](#)).⁵ The literature has also found that output-dependent prizes can mitigate peer-to-peer sabotage and encourage helping ([Danilov, Harbring, and Irlenbusch, 2019](#); [Glökler, Pull, and Stadler, 2022](#)) and that handicaps can exacerbate peer-to-peer sabotage ([Brown and Chowdhury, 2017](#)). By contrast, we consider a setting where the manager has the discretion to hire a new employee to compete against, and the manager may sabotage the *hiring* process to prevent competition from the new hire. While managerial authority in hiring and promotion has been explored in organizational economics (e.g., [Carmichael \(1988\)](#); [Lazear \(1995\)](#); [Dessein \(2002\)](#); [Friebel and Raith \(2004\)](#); [Chen \(2024\)](#); [Haegele \(2025\)](#)), its implications

⁴In Appendix B, we explore an extension where the loser also receives a constant share of total output. We show that our main results remain robust: the optimal contract assigns a zero share to the loser, as giving a head start to the manager is more effective at mitigating hiring sabotage.

⁵See [Chowdhury and Gürtler \(2015\)](#) for a survey of (peer-to-peer) sabotage in tournaments.

for tournaments remain unexplored.

The literature has considered the use of head starts or handicaps in tournaments to mitigate heterogeneity in players’ abilities (e.g., Lazear and Rosen (1981); O’Keeffe, Viscusi, and Zeckhauser (1984)). The common wisdom is to give a head start to the weaker player (i.e., handicap the stronger player) to “level the playing field,” as player heterogeneity would diminish incentives for both players. Recent papers have challenged this common wisdom. Drugov and Ryvkin (2017) show that head starts or handicaps can be optimal even when players have the same ability. Fu and Wu (2020) and Drugov and Ryvkin (2022) find that giving a head start to the stronger player can increase total effort. Consistent with these findings, we find that in Lazear-Rosen-style tournaments with output-dependent prizes, a head start to the stronger player can increase total effort. To the best of our knowledge, the literature has not explored tournaments where an agent has the discretion to hire another, *a fortiori* the use of head starts to mitigate sabotage in this process.

2 The Model

2.1 Setup

Consider a two-player tournament in which a player, the manager m , has the discretion to hire the other agent n . This framework captures situations where an organization operating under tournament incentives needs to hire a new agent, but the principal cannot observe the abilities of the candidates and must delegate the hiring decision to a manager, who may also be interpreted as a hiring committee. The manager of ability (type) $\theta_m \in \Theta \equiv [\underline{\theta}, \bar{\theta}]$ observes the abilities of the candidates and selects a new hire of ability $\theta_n \in \Theta$. We assume that $\underline{\theta} > 0$ is sufficiently small. Once the hiring decision is made, the manager competes with the new hire in a Lazear-Rosen-style tournament. In the tournament, both agents have common knowledge of their abilities (θ_m, θ_n) , and agent $i \in \{m, n\}$ invests effort $e_i \geq 0$ towards production at a cost $c(e_i)/\theta_i$. Assume that $c(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly convex, with $c(0) = c'(0) = 0$ and $\lim_{e \rightarrow \infty} c'(e) > 2\bar{\theta}$. The output of agent i is given by $y_i = e_i + \varepsilon_i$, where the term ε_i captures a zero-mean random shock, which can also be interpreted as noise in performance evaluation. We assume that the tournament prize $V(y)$ depends on the total output $y = y_m + y_n$, instead of a fixed prize, in order to provide incentives for the manager to hire a high-ability agent.⁶ For tractability, we further assume the tournament prize is a constant share of total output, i.e., $V(y) = \alpha y$, which can be interpreted as equity compensation, such as firm stock or promotion to partnership.⁷ The profit-

⁶If the tournament prize does not depend on total output, the manager will always hire the lowest-ability agent.

⁷See Gershkov, Li, and Schweinzer (2009) and Güth et al. (2016) for prizes as a constant share of total output in tournaments. In a Lazear-Rosen-style tournament without managerial discretion, Güth et al. (2016) show that this prize structure is more cost-effective than a fixed prize. Managerial discretion makes the output-dependent prize even more

maximizing principal can commit to the payout ratio $\alpha \in [0, 1]$ through formal equity compensation contracts or through repeated interaction and reputation. In addition, she can also offer a *head start* $h \in [-\bar{H}, \bar{H}]$ to the manager—an advantage in output comparison—to incentivize her to hire a higher-ability agent.⁸ In practice, the firm may reward better-performing agents with stock or promotion and offer a seniority premium in performance evaluations. We assume that the bound $\bar{H} > 0$ on the head start is large. Given the head start h , the manager wins the prize if and only if $y_m + h \geq y_n$.

Alternatively, the tournament scheme can be viewed as a sharing contract: the principal retains a fraction $1 - \alpha$ of the total output, and the remaining fraction α constitutes a compensation pool $V = \alpha y$ shared by the agents. Specifically, each agent’s percentage share of the compensation pool in the sharing contract equals their probability of winning in the tournament, which is *endogenously* determined by their effort levels. A head start $h > 0$ increases the manager’s percentage share of the compensation pool and reduces the new hire’s percentage share.

The timing of the game is as follows. First, the principal knows the manager’s ability and commits to the contract (α, h) . Next, the manager chooses an agent $\theta_n \in \Theta$ from the candidate pool. Then, the manager and the new hire choose their effort levels in the tournament, where abilities are common knowledge to them. Finally, the output is realized, and agents are paid according to the contract (α, h) . The solution concept we use is the pure-strategy subgame-perfect Nash equilibrium (SPNE).

2.2 Tournament Stage

Given the head start h , the manager m will outperform n with probability

$$\Pr(y_m + h \geq y_n) = \Pr(e_m - e_n + h \geq \varepsilon_n - \varepsilon_m).$$

Assume that the difference in random shocks, $\varepsilon_n - \varepsilon_m$, follows a distribution $G(\cdot)$ with a continuously differentiable density $g(\cdot)$ that is unimodal and symmetric around zero (i.e., $g(x) = g(-x)$). For example, the distribution can be generated by independent and identically distributed (i.i.d.) random shocks with a unimodal distribution (Purkayastha (1998, Theorem 2.2)). Therefore, the probability that m will outperform n is $\Pr(y_m + h \geq y_n) = G(e_m - e_n + h)$. By symmetry, the probability that n will outperform m is $G(e_n - e_m - h) = 1 - G(e_m - e_n + h)$. We also assume $\mathbf{E}[\varepsilon_n + \varepsilon_m \mid \varepsilon_n - \varepsilon_m] = 0$ so that the expected total output is the same regardless of the winner’s identity, which is satisfied if the random shocks are i.i.d. and symmetric around zero.

desirable because it incentivizes the manager to hire a high-ability agent.

⁸We allow for the possibility that $h < 0$, which makes it a *handicap*—a bias *against* the manager when comparing outputs. In the absence of the manager’s discretion, it is optimal to give a head start to the higher-ability agent in our setting (see Remark 3).

Additionally, we parameterize the distribution of the difference in random shocks by $G(x) = F(x/\sigma)$, where F is a CDF with unit variance, and $\sigma > 0$ is a scale parameter that measures the noise in performance evaluation (i.e., the dispersiveness and the variance of G).⁹ This distribution can be generated by i.i.d. random shocks that belong to a location-scale family with variance $\sigma^2/2$ and mean zero (Drugov and Ryvkin, 2020; Morgan, Tumlinson, and Várdy, 2022) as well as correlated random shocks within a location-scale family, as shown in the following examples.

Example 1 (Normal Distribution). If $\varepsilon_m, \varepsilon_n \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2/2)$, then $\varepsilon_n - \varepsilon_m \sim \mathcal{N}(0, \sigma^2)$.

Example 2 (Logistic Distribution). If $\varepsilon_m, \varepsilon_n \stackrel{\text{i.i.d.}}{\sim} \text{Gumbel}(0, s)$, then $\varepsilon_n - \varepsilon_m \sim \text{Logistic}(0, s)$. Under this distribution, the tournament is equivalent to a Tullock (1980) contest with discriminatory power $r = 1/s$ and a multiplicative head start (after the log transformation of efforts).

Example 3 (Uniform Distribution). If $\varepsilon_m, \varepsilon_n \sim \text{Unif}[-\frac{M}{4}, \frac{M}{4}]$ and $\varepsilon_m = -\varepsilon_n$, then $\varepsilon_n - \varepsilon_m \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$, and $\mathbf{E}[\varepsilon_n + \varepsilon_m \mid \varepsilon_n - \varepsilon_m] = 0$ also holds.¹⁰

We use backward induction to derive the pure-strategy SPNE. Given the new hire's ability θ_n , in the subgame $\Gamma(\theta_n)$ (i.e., the tournament stage), the expected payoffs for m and n are¹¹

$$u_m(e_m, e_n, \theta_m) = \alpha \cdot G(e_m - e_n + h)(e_n + e_m) - \frac{c(e_m)}{\theta_m}, \quad (1)$$

$$u_n(e_m, e_n, \theta_n) = \alpha \cdot G(e_n - e_m - h)(e_n + e_m) - \frac{c(e_n)}{\theta_n}. \quad (2)$$

Both agents will choose the effort levels (e_m, e_n) to maximize their expected payoffs. Since each agent can guarantee a nonnegative payoff by choosing zero effort, effort levels with average cost $c(e_i)/e_i > 2\bar{\theta}$ cannot arise in equilibrium. Consequently, the equilibrium effort levels are bounded (see Lemma A.1). The first-order conditions yield

$$\frac{\partial u_m}{\partial e_m} = \alpha[(e_n + e_m) \cdot g(e_m - e_n + h) + G(e_m - e_n + h)] - \frac{c'(e_m)}{\theta_m} = 0, \quad (3)$$

$$\frac{\partial u_n}{\partial e_n} = \alpha[(e_n + e_m) \cdot g(e_n - e_m - h) + G(e_n - e_m - h)] - \frac{c'(e_n)}{\theta_n} = 0. \quad (4)$$

Following the standard practice in the tournament literature, we assume the variance of G is sufficiently large (or the cost function is sufficiently convex) so that the second-order conditions

⁹After the parameterization, the distribution G has a larger variance if and only if the difference in random shocks is more dispersed (Drugov and Ryvkin (2020, Proposition 2)). When random shocks are i.i.d. and within a location-scale family, their difference is more dispersed if and only if the random shocks themselves are more dispersed. More generally, when random shocks are i.i.d. and log-concave, their difference is more dispersed if the shocks themselves are more dispersed (Shaked and Shanthikumar (2007, Theorem 3.B.9)).

¹⁰If ε_m and ε_n are i.i.d. uniform random variables, then $\varepsilon_n - \varepsilon_m$ follows a symmetric triangular distribution.

¹¹Note that the subgame is not supermodular in (e_m, e_n) unless $g'(x) = 0$ (uniform distribution).

are satisfied, and a pure-strategy equilibrium exists.¹² Thus, the two equations above determine the equilibrium effort levels $(e_m(\theta_n), e_n(\theta_n))$ in the subgame as functions of the hiring decision θ_n . The following lemma provides the comparative statics.

Lemma 1. *Given θ_n , the equilibrium effort in the subgame $\Gamma(\theta_n)$ is given by equations (3) and (4). The following comparative statics hold.*

(i) *The manager's effort $e_m(\theta_n)$ is*

- *strictly increasing in the head start h , the payout ratio α , and her ability θ_m ;*
- *decreasing in the noise in performance evaluation σ if $h \geq 0$;¹³*
- *increasing (decreasing) in the new hire's ability θ_n if $\theta_m \geq \theta_n$ and $h \geq 0$ ($\theta_n \geq \theta_m$ and $h \leq 0$).*

(ii) *The new hire's effort $e_n(\theta_n)$ is*

- *strictly decreasing in the head start h , and strictly increasing in α and θ_n ;*
- *decreasing in σ if $h \leq 0$;*
- *increasing (decreasing) in θ_m if $\theta_n \geq \theta_m$ and $h \leq 0$ ($\theta_m \geq \theta_n$ and $h \geq 0$).*

(iii) *There exists $\bar{h}_1, \bar{h}_2 \geq 0$ such that if $h > \bar{h}_1$ ($h < -\bar{h}_2$), then $e_m(\theta_n)$ is increasing (decreasing) in θ_n , and $e_n(\theta_n)$ is decreasing (increasing) in θ_m .¹⁴*

Remark 1. Because the abilities of both agents are fixed in this lemma, the results extend the literature on head starts (or handicaps) in tournaments with fixed prizes to a setting with output-dependent prizes. The effects of the head start and noise are consistent with this literature (Cf. O'Keeffe, Viscusi, and Zeckhauser (1984); Drugov and Ryvkin (2020)). To our knowledge, the comparative statics of individual efforts with respect to the head start, noise, and abilities, which may be interdependent, are novel in this setting with output-dependent prizes.

The proof is in Appendix A. Given the new hire's ability θ_n , a head start h increases the manager's chance of winning, encouraging the manager to invest more effort while discouraging

¹²See Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), and Ryvkin and Drugov (2020). When x is bounded and $g(x)$ is twice continuously differentiable, a large variance implies that $g(x)$ and $g'(x)/g(x)$ are sufficiently small (see Lemma A.2). For example, the normal distribution $\mathcal{N}(0, \sigma^2)$ has $g(x) \leq \frac{1}{\sqrt{2\pi}\sigma}$ and $g'(x)/g(x) = -x/\sigma^2$.

¹³The necessary and sufficient condition is that $h > -2e_m(\theta_n)$, i.e., the head start is not too against her. Symmetrically, the new hire's effort is decreasing in σ if and only if $h < 2e_n(\theta_n)$.

¹⁴The more precise condition is that the manager is more (less) likely to win; the lemma provides two sufficient conditions. For example, $\frac{de_m}{d\theta_n}$ is positive (negative) if $g'(e_m^* - e_n^* + h)$ is negative (positive), which holds when the manager is more (less) likely to win because $g(x)$ is unimodal. For strictly unimodal $g(x)$, the condition is necessary and sufficient. For the uniform distribution, $\frac{de_m}{d\theta_n} = 0$ because $g'(x) \equiv 0$ on the support.

the new hire from doing so. We will explore these two effects in detail in the next section, where we also show that, because of the output-dependent prizes, it can be suboptimal to give a head start to the lower-ability agent (even in the absence of the managerial discretion), in contrast to the common wisdom of “leveling the playing field” (see Lemma 3).

As the payout ratio α increases, the stakes in the tournament rise, providing stronger incentives for both the manager and the new hire to invest effort. Thus, both agents invest more effort as the payout ratio increases.

Within a location-scale family of the distribution G , as the scale parameter σ (or the variance σ^2) increases, the difference in random shocks becomes more dispersed, and the performance evaluation becomes noisier. Therefore, the manager and the new hire have weaker incentives to invest effort when the competition is fair (i.e., $h = 0$) or the head start is in their favor. However, the effect on effort is reversed if the head start is sufficiently against them. For example, when the head start $h > 0$ is sufficiently large, a noisier evaluation encourages the new hire to invest more effort because it gives him some hope of winning despite the large bias against them.

Finally, as the manager’s ability θ_m increases, she invests more effort due to a lower marginal cost of effort. At the same time, the new hire is incentivized to invest more effort to outperform the manager if he is more likely to win than the manager. Otherwise, if the new hire is less likely to win, he will invest less effort when the manager’s ability increases. Technically, this is because the new hire’s payoff is supermodular (submodular) in her effort and the manager’s effort if the new hire is more (less) likely to win. By symmetry, as the new hire’s ability θ_n increases, he invests more effort, whereas the manager invests more (less) effort if she is more (less) likely to win. Intuitively, the manager is more (less) likely to win when her ability is high (low) compared to the new hire’s or when the head start is large (small).

The following example illustrates these results under the uniform distribution and quadratic costs.

Running Example (Uniform–Quadratic). Suppose $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, with M sufficiently large so that all winning probabilities are interior. The equilibrium effort levels in the subgame $\Gamma(\theta_n)$ are given by

$$\begin{aligned} e_m(\theta_n) &= \frac{M/2 + h}{M/\alpha\theta_m - 2}, \\ e_n(\theta_n) &= \frac{M/2 - h}{M/\alpha\theta_n - 2}. \end{aligned}$$

Under the uniform distribution, an agent’s marginal benefit from increasing effort is independent of the opponent’s effort level. Consequently, strategic interdependence in effort choices disappears,

and agents' equilibrium effort levels do not depend on their opponents' abilities.¹⁵ The effort levels are increasing in the amount of head start in their favor, the payout ratio, and their own abilities. Moreover, both the manager's and the new hire's efforts decrease in the variance of G if and only if the head start is not too against them—i.e., $h \geq -\alpha\theta_m$ and $h \leq \alpha\theta_n$ respectively.

2.3 Hiring Stage

At the hiring stage, the manager chooses $\theta_n \in \Theta$ to maximize her expected payoff in the subgame $\Gamma(\theta_n)$. By Lemma 1, because both $c(\cdot)$ and $g(\cdot)$ are continuously differentiable, $e_n(\theta_n)$ is continuously differentiable and increasing in θ_n . Therefore, we have

$$\frac{du_m(e_m(\theta_n), e_n(\theta_n))}{d\theta_n} = \frac{\partial u_m}{\partial e_n} e'_n(\theta_n) = \alpha[G(e_m - e_n + h) - (e_m + e_n) \cdot g(e_m - e_n + h)]e'_n(\theta_n). \quad (5)$$

When the manager hires a higher-ability agent, the agent will invest more effort at the tournament stage, which increases the size of the tournament prize but also reduces the manager's probability of winning. The new hire's ability affects the manager's payoff only through his own effort (e_n), and not through the manager's effort, as e_m is chosen optimally by the manager at the tournament stage. Therefore, the manager chooses θ_n to determine the new hire's effort $e_n(\theta_n)$ in the subgame and maximize her expected payoff.

Hence, in the SPNE, when the hiring decision $\theta_n^* \in (\underline{\theta}, \bar{\theta})$, the first-order conditions of θ_n^* and the equilibrium effort levels (e_m^*, e_n^*) are given by

$$G(e_m^* - e_n^* + h) = (e_m^* + e_n^*) \cdot g(e_m^* - e_n^* + h) \quad (6)$$

$$c'(e_n^*)/\theta_n^* = \alpha, \quad (7)$$

$$c'(e_m^*)/\theta_m = 2\alpha \cdot G(e_m^* - e_n^* + h). \quad (8)$$

The optimal hiring decision θ_n^* balances the marginal benefit of hiring a higher-ability agent (i.e., a higher tournament prize) and the marginal cost (i.e., a lower probability of winning), as determined by equation (6). Given his ability θ_n^* chosen optimally by the manager, the first-order condition of the new hire's effort collapses to equation (7): in the equilibrium, his marginal cost of effort equals its marginal effect on the expected tournament prize (i.e., $\partial \mathbf{E}[V]/\partial e_n = \alpha$). On the other hand, the manager's marginal cost of effort is higher (lower) than its marginal effect on the expected prize if she is more (less) likely to win the tournament.

In general, it is not possible to obtain a closed-form solution. When the hiring decision θ_n^* takes

¹⁵With a uniform distribution, it is no longer necessary to assume that abilities are common knowledge to both agents in the tournament, allowing the possibility that the new hire does not know the manager's ability.

the corner solution $\theta_n^* \in \{\underline{\theta}, \bar{\theta}\}$, the comparative statics in Lemma 1 apply. For an interior solution, we obtain the following results.

Lemma 2. *Assume $\theta_n^* \in (\underline{\theta}, \bar{\theta})$. The equilibrium effort levels (e_m^*, e_n^*) and hiring decision θ_n^* are given by equations (6)–(8). The following comparative statics hold.*

(i) *The manager's effort e_m^* is*

- *strictly increasing in the head start h , the payout ratio α , and her ability θ_m ;*
- *strictly decreasing in the noise in performance evaluation σ if $h \geq 0$.¹⁶*

(ii) *The hiring decision θ_n^* is strictly increasing in h and σ , and strictly decreasing in α .*

(iii) *The new hire's effort e_n^* is strictly increasing in h and σ .*

(iv) *There exists $\bar{h}_1, \bar{h}_2 > 0$ such that if $h > \bar{h}_1$ ($h < -\bar{h}_2$), then*

- *θ_n^* is increasing (decreasing) in θ_m ;*
- *e_n^* is increasing (decreasing) in θ_m and α .¹⁷*

(v) *There exists $\bar{\theta}_m, \underline{\theta}_m \in \Theta$ such that if $\theta_m > \bar{\theta}_m$ ($\theta_m < \underline{\theta}_m$) and $h \geq 0$ ($h \leq 0$), then*

- *θ_n^* is increasing (decreasing) in θ_m ;*
- *e_n^* is increasing (decreasing) in θ_m and α .*

A higher head start encourages the manager to invest more effort. Additionally, increasing the head start further insulates her from competition, leading her to hire a higher-ability agent until the strongest candidate is hired. The head start affects the new hire's effort through two channels: on the one hand, it discourages effort by lowering his chance of winning; on the other, it leads to a higher-ability hire, which in turn increases his effort. Overall, the net effect on the new hire's effort is positive. We will discuss them in detail in the next section.

As the difference in random shocks become more dispersed (captured by a higher σ), performance evaluation becomes noisier, so the manager has weaker incentives to invest effort when the head start is in her favor ($h \geq 0$). A noisier performance evaluation also makes the manager less concerned about competing with a higher-ability agent, leading her to hire a higher-ability candidate. The variance affects the new hire's effort through two channels: on the one hand, it discourages effort by making performance evaluation noisier; on the other, it results in a higher-ability new hire, which in turn increases his effort. Overall, the net effect on the new hire's effort is positive.

¹⁶The necessary and sufficient condition is that the head start is not too against the manager, i.e., $h > -2e_m^*$.

¹⁷The more precise condition for each statement in (iv) and (v) is that the manager is more (less) likely to win, which is necessary and sufficient for strictly unimodal $g(x)$.

As the payout ratio α increases, the stakes in the tournament rise, providing stronger incentives for the manager to invest effort. The payout ratio also affects the new hire's effort through two channels. On the one hand, it directly increases the new hire's incentives to invest effort. On the other hand, it increases the manager's incentive to prevent competition by hiring a lower-ability agent, especially when she is less likely to win. Overall, the net effect on the new hire's effort is positive (negative) if the manager is more (less) likely to win. The manager is more (less) likely to win if his ability is sufficiently high (low) and the head start is in favor of (against) her.

Similarly, as the manager's ability θ_m increases, she will invest more effort because her marginal cost of effort is lower. The manager's ability also affects the new hire's effort through two channels: if the manager is more likely to win, it incentivizes the new hire to exert more effort to outperform the manager; on the other hand, a higher-ability manager chooses a higher-ability agent, which further increases the new hire's effort. Overall, the net effect on the new hire's effort is positive (negative) if the manager is more (less) likely to win.

The following example illustrates the results with uniform distribution and quadratic cost.

Running Example (Uniform–Quadratic). Suppose $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, with M sufficiently large so that all winning probabilities are interior. The equilibrium hiring decision and effort levels and are given by

$$\begin{aligned}\theta_n^* &= \min\left(\frac{h + M/2}{2\alpha}, \bar{\theta}\right), \\ e_m^* &= \frac{M/2 + h}{M/\alpha\theta_m - 2}, \\ e_n^* &\equiv e_n(\theta_n^*) = \min\left(\frac{h + M/2}{2}, \frac{M/2 - h}{M/\alpha\bar{\theta} - 2}\right).\end{aligned}$$

The uniform assumption removes the strategic interdependence of effort choices in the tournament stage (because $g'(x) = 0$), so neither the new hire's effort e_n^* nor his ability θ_n^* depends on the manager's ability θ_m . Furthermore, the new hire's effort e_n^* is also independent of the payout ratio α when $\theta_n^* < \bar{\theta}$ because its positive effect on the new hire's effort is exactly offset by its negative effect through the decrease in the new hire's ability due to increased managerial sabotage.

3 Optimal Head Start

The effects of the head start on the expected total output $\mathbf{E}[y^*] = e_m^* + e_n^*$ can be decomposed into three components:

$$\frac{d\mathbf{E}[y^*]}{dh} = \frac{de_m^*}{dh} + \left.\frac{\partial e_n(\theta_n, h)}{\partial h}\right|_{\theta_n=\theta_n^*(h)} + \left.\frac{\partial e_n(\theta_n, h)}{\partial \theta_n}\right|_{\theta_n=\theta_n^*(h)} \theta_n^{*'}(h).$$

According to Lemmas 1 and 2, we identify the three effects and their directions as follows:

1. Encouragement effect on e_m : $\frac{de_m^*}{dh} > 0$.
2. Discouragement effect on e_n : $\frac{\partial e_n(\theta_n^*(h), h)}{\partial h} < 0$.
3. Hiring effect on e_n (through θ_n^*): $\frac{\partial e_n(\theta_n^*(h), h)}{\partial \theta_n} \theta_n^{*'}(h) \geq 0$ (> 0 if $\theta_n^*(h) < \bar{\theta}$).

Intuitively, the head start increases the manager's probability of winning. Since the tournament prize is increasing in effort, it has an *encouragement effect* on the manager by increasing her marginal return to effort. However, the impact of the head start on the new hire's effort is rather mixed. On the one hand, holding his ability θ_n fixed, the head start reduces his marginal return to effort, thereby having a *discouragement effect*. On the other hand, the head start partially insulates the manager from competition and leads the manager to hire a higher-ability agent. This *hiring effect* results in greater effort from the higher-ability new hire.

Importantly, by Lemma 2, the hiring effect dominates the discouragement effect until the highest-ability agent is hired (i.e., when $\theta_n^*(h) < \bar{\theta}$) because

$$\frac{de_n^*}{dh} = \underbrace{\frac{\partial e_n(\theta_n, h)}{\partial h} \Big|_{\theta_n = \theta_n^*(h)}}_{\text{Discouragement effect}} + \underbrace{\frac{\partial e_n(\theta_n, h)}{\partial \theta_n} \Big|_{\theta_n = \theta_n^*(h)} \theta_n^{*'}(h)}_{\text{Hiring effect}} > 0$$

Thus, the head start always increases total output until the strongest candidate is hired, so the optimal head start must (at least) induce the manager to hire the highest-ability agent $\bar{\theta}$.

Running Example (Uniform–Quadratic). Assume $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$. When $\theta_n^*(h) < \bar{\theta}$, the encouragement and discouragement effects are given by

$$\frac{de_m^*}{dh} = \frac{1}{M/\alpha\theta_m - 2} > 0, \quad \frac{\partial e_n(\theta_n^*, h)}{\partial h} = -\frac{1}{M/\alpha\theta_n^* - 2} < 0$$

respectively. The sum of the discouragement and the hiring effects is $\frac{de_n^*}{dh} = \frac{1}{2}$.

Because $\theta_n^*(h)$ is strictly increasing in h until the strongest candidate is hired (i.e., $\theta_n^*(h) = \bar{\theta}$), we denote by $\bar{h}(\alpha)$ the head start just enough to induce the manager to hire the highest-ability candidate—i.e., $\theta_n^*(h) = \bar{\theta}$ for all $h \geq \bar{h}(\alpha)$ and $\theta_n^*(h) < \bar{\theta}$ for all $h < \bar{h}(\alpha)$. In general, by Lemma 2, $\bar{h}(\alpha)$ is increasing in α and decreasing in the variance of G . This is because a higher payout ratio α or a lower noise in the output evaluation increases the manager's incentives to sabotage, and therefore requires a higher head start to induce the manager to hire the strongest candidate.

Running Example (Uniform–Quadratic). For $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, the head start just enough to induce the manager to hire the strongest candidate is $\bar{h}(\alpha) = 2\alpha\bar{\theta} - M/2$.

Now we have already shown that the optimal head start is at least $\bar{h}(\alpha)$, a natural question is whether it is desirable to give the manager a further head start. As θ_n cannot increase beyond $\bar{\theta}$, a further head start will no longer have the hiring effect. Hence, we need to compare the discouragement effect (on the new hire) and the encouragement effect (on the manager) in the absence of the hiring effect (i.e., holding θ_n fixed). The following lemma provides a necessary and sufficient condition when the discouragement effect dominates the encouragement effect.

Lemma 3. *Given θ_n , the expected total output ($e_m^* + e_n^*$) is decreasing in h if and only if*

$$(e_m^* + e_n^*) \frac{g'(e_m^* - e_n^* + h)}{g(e_m^* - e_n^* + h)} \leq \frac{c''(e_m^*)/\theta_m - c''(e_n^*)/\theta_n}{c''(e_m^*)/\theta_m + c''(e_n^*)/\theta_n}. \quad (9)$$

Remark 2. For quadratic costs $c(e) = e^2/2$, this condition simplifies to

$$(e_m^* + e_n^*) \frac{g'(e_m^* - e_n^* + h)}{g(e_m^* - e_n^* + h)} \leq \frac{\theta_n - \theta_m}{\theta_n + \theta_m},$$

which holds if $\theta_n > \theta_m$ and the variance is sufficiently large. For uniform distribution and quadratic costs, the condition is equivalent to $\theta_n \geq \theta_m$.

For normal distribution $G \sim \mathcal{N}(0, \sigma^2)$ and $c(e) = e^2/2$, the condition is equivalent to

$$(e_m^* + e_n^*)(e_n^* - e_m^* - h) \leq \frac{\theta_n - \theta_m}{\theta_n + \theta_m} \sigma^2,$$

which holds for all $h \geq \bar{h}(\alpha)$ if $\theta_n > \theta_m$ and σ^2 is large.

Remark 3. Lemma 3 also implies that, even without the managerial discretion (i.e., when abilities are exogenous), it can be suboptimal to give a head start to the lower-ability agent in tournaments with output-dependent prizes, in contrast to the common wisdom of “leveling the playing field.”

Indeed, when $\theta_n = \bar{\theta}$ is fixed, handicapping the manager instead may generate higher total output. For each α , the loss in profit from choosing the head start $\bar{h}(\alpha)$ rather than the output-maximizing head start for fixed agent types captures the opportunity cost of delegating the hiring decision to the manager.¹⁸

By Lemma 3, if the regularity condition (9) holds for all $h > \bar{h}(\alpha)$, the discouragement effect dominates the encouragement effect when abilities are exogenous. Therefore, any further head start beyond $\bar{h}(\alpha)$ decreases total effort.

¹⁸Suppose that $c'''(\cdot) \geq 0$, $g(\cdot)$ is log-concave, and the variance is sufficiently large so that the comparative statics in Lemma 1 hold. Then, Lemma 3 implies that there exists a cutoff \tilde{h} (possibly equal to $-\bar{H}$) such that condition (9) holds if and only if $h \geq \tilde{h}$. Thus, when abilities are fixed, \tilde{h} characterizes the output-maximizing head start.

Running Example (Uniform–Quadratic). If $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, condition (9) is satisfied if $\theta_n \geq \theta_m$. Therefore, once the highest-ability agent $\theta_n = \bar{\theta}$ is hired by the manager, any further head start given to the manager will decrease total output because the discouragement effect dominates the encouragement effect, that is,

$$\frac{d(e_m^* + e_n^*)}{dh} = \underbrace{\frac{1}{M/\alpha\theta_m - 2}}_{\text{Encouragement effect}} - \underbrace{\frac{1}{M/\alpha\bar{\theta} - 2}}_{\text{Discouragement effect}} \leq 0 \text{ when } h > \bar{h}(\alpha).$$

Thus, the optimal head start is $\bar{h}(\alpha) = 2\alpha\bar{\theta} - M/2$.

Consequently, Lemmas 2 and 3 imply the following proposition on the optimal head start.

Proposition 1. *For any given $\alpha \in (0, 1)$, the optimal head start ensures that the manager hires the highest-ability agent. Moreover, if condition (9) holds for all $h > \bar{h}(\alpha)$ and $\theta_n = \bar{\theta}$, the optimal head start is exactly $\bar{h}(\alpha)$, which is just enough to induce the manager to hire the highest-ability agent.*

Remark 4. We have assumed that the bound \bar{H} on the head start is sufficiently large so that a head start of $\bar{h}(\alpha)$ is always feasible. Otherwise, if large head starts are infeasible due to fairness concerns, the optimal head start is $\min(\bar{h}(\alpha), \bar{H})$, and hiring sabotage may still occur.

In words, the optimal head start always ensures the manager hires the strongest candidate and thus eliminates hiring sabotage. Furthermore, under a regularity condition that is satisfied when the variance is sufficiently large, the optimal head start is exactly at the level that induces the manager to hire the strongest candidate.

4 Optimal Payout Ratio

In the previous section, we have shown that under some assumptions, for any given $\alpha \in (0, 1)$, the optimal head start is $\bar{h}(\alpha)$, which is just enough to ensure the manager hires the strongest candidate. The principal also faces a trade-off when determining the optimal payout ratio: a higher payout ratio reduces the principal's share of total output but may incentivize agents to exert more effort. According to Lemma 2, the payout ratio affects the manager's and the new hire's effort in two ways. On the one hand, a higher payout ratio encourages them to exert more effort. On the other hand, it affects the new hire's ability because the manager may hire a higher or lower-ability agent.

Given the optimal head start $\bar{h}(\alpha)$, the optimal payout ratio is given by

$$\alpha^* \in \arg \max_{\alpha} \Pi(\alpha), \quad \text{where } \Pi(\alpha) = (1 - \alpha)(e_m^*(\alpha, \bar{h}(\alpha)) + e_n^*(\alpha, \bar{h}(\alpha))). \quad (10)$$

While an interior maximizer $\alpha^* \in (0, 1)$ exists, its analytical solution and comparative statics are intractable, as the parameters $(\alpha, \sigma, \theta_m)$ affect efforts through both directly and through $\bar{h}(\alpha)$.¹⁹ For tractability, we henceforth assume uniform distribution $G \sim \text{Unif}[-M/2, M/2]$ and quadratic cost $c(e) = e^2/2$ to remove the strategic interdependence of effort choices in the tournament stage (see also Konrad (2009), Ederer (2010), and Brown and Minor (2014)).

Given the optimal head start $\bar{h}(\alpha) = 2\alpha\bar{\theta} - M/2$, an increase in the payout ratio α has a positive effect on the effort levels of both agents. From the new hire's perspective, since $\theta_n^* = \bar{\theta}$, an increase in α incentivizes his effort as $e_n^*(\alpha, \bar{h}(\alpha)) = \alpha\bar{\theta}$. From the manager's perspective, an increase in α creates a greater incentive for the manager to both exert effort and engage in sabotage. The higher incentive to sabotage leads to an increase in the optimal head start, which further incentivizes the manager because of the encouragement effect of the head start, as $e_m^*(\alpha, \bar{h}(\alpha)) = \frac{2\alpha\bar{\theta}}{M/\alpha\theta_m - 2}$.

Proposition 2. *Under the uniform–quadratic assumption with $M > (1 + \sqrt{2})\bar{\theta}$, the optimal payout ratio is $\alpha^* = \left(1 + \sqrt{1 - 2\frac{\theta_m}{M}}\right)^{-1}$ and the optimal head start is $h^* = 2\alpha^*\bar{\theta} - M/2$, which is just enough to ensure the manager hires the highest-ability agent. The principal's profit is $\Pi^* = \left(2\left(\frac{1}{\alpha^*} - \frac{\theta_m}{M}\right)\right)^{-1}\bar{\theta}$.²⁰*

Corollary 2.1. *Under the uniform–quadratic assumption, the following comparative statics hold:*

- (i) *The optimal head start h^* is decreasing in M and increasing in the manager's ability θ_m , and $h^* > 0$ if and only if $M < \frac{8}{(4\bar{\theta} - \theta_m)}$.*
- (ii) *The optimal payout ratio α^* is decreasing in M and increasing in the manager's ability θ_m . As $M \rightarrow \infty$, $\alpha^* \rightarrow 0.5$. As $M \rightarrow 2\bar{\theta}$, $\alpha^* \rightarrow \left(1 + \sqrt{1 - \theta_m/\bar{\theta}}\right)^{-1}$. As $\theta_m \rightarrow 0$, $\alpha^* \rightarrow 0.5$.*
- (iii) *The optimal profit Π^* is decreasing in M and increasing in the manager's ability θ_m .*

As we can see, the optimal payout ratio α^* and head start h^* depend on the manager's ability θ_m and the noise in the performance evaluation captured by M . As the manager's ability θ_m increases, the manager invests more effort because his marginal cost of effort decreases, so the principal's profit is higher. Meanwhile, when θ_m is higher, the payout ratio also has a greater marginal effect on the manager's effort, so the optimal payout ratio α^* is higher. The increase in payout ratio creates a greater incentive to sabotage, thereby requiring the principal to increase the optimal head start h^* to ensure that the new hire is of the highest ability.

In equilibrium, the principal offers a head start to the manager that ensures the highest-ability agent is hired. As the noise in performance evaluation increases, the noise begins to take over

¹⁹The existence of an interior maximizer follows from the continuity of $\Pi(\alpha)$ on $[0, 1]$ (because $e_m^*(\alpha, h)$, $e_n^*(\alpha, h)$, and $\bar{h}(\alpha)$ are all continuous in α and h), $\Pi(0) = \Pi(1) = 0$, and $\Pi(\alpha) > 0$ for all $\alpha \in (0, 1)$.

²⁰To ensure the pure-strategy equilibrium exists, the winning probability $G(e_m^* - e_n^* + h^*)$ has to be interior, and each agent receives a nonnegative payoff. By Proposition 1, a sufficient condition is that $M > (1 + \sqrt{2})\bar{\theta}$.

performance attribution. As a result, the manager becomes less fearful of competing against a higher-ability candidate. Thus, the manager has a lower incentive to sabotage, which allows the principal to reduce the optimal head start h^* .

Moreover, for a given *optimal* head start, the increase in noise makes the effort less important in determining the winner due to noise in performance measurement, thereby reducing the marginal effect of the payout ratio on the manager's effort. In other words, for a marginal increase in payout ratio, the manager's effort increases less than it would if noise were lower. Therefore, under higher noise, the principal will lower the optimal payout ratio α^* , which in turn decreases the optimal head start even further. Furthermore, higher noise also reduces the manager's marginal return to effort and thus has a demotivating effect on his equilibrium effort, thereby adversely impacting the principal's profit.

5 Conclusion

The tournament model is an attractive choice for firms seeking to motivate employees while minimizing monitoring costs. Professional services organizations—such as accounting, law, consulting, and investment firms—often operate as tournament-based partnerships, where a finite number of partnership slots distinguish these systems from other relative performance incentive schemes (Lazear, 1995).

Existing tournament literature does not consider cases where one player can choose the ability of another. We fill this gap by studying the optimal design of a two-player Lazear-Rosen-style tournament, where the manager has discretion over hiring the other player, and the winner receives a fraction of total output. Tournament incentives may lead hiring managers to hire a lower-ability agent than would otherwise be optimal for the firm to avoid future competition.

To mitigate hiring sabotage, the principal designs a head start—an advantage to the manager in the output comparison—and a payout ratio, which is the share of total output awarded to the winner. This role of head starts builds on the literature on biased contests, where head starts (or handicaps) are used to restore efficiency or provide incentives in tournaments. The output-dependent prize makes the new hire also a coworker and provides some incentives for the manager to hire a high-ability candidate.

We find the head start has three effects on the output: (i) encouragement effect on the manager, (ii) discouragement effect on the new hire, and (iii) hiring effect through the increased ability of the new hire. The hiring effect dominates the discouragement effect until the strongest candidate is hired; once the strongest candidate is hired, any further head start leads the discouragement effect to dominate the encouragement effect. Therefore, the optimal contract offers just enough head start to induce the manager to hire the strongest candidate. Consequently, we derive the optimal head start

and payout ratio that maximize the principal's profit.

In reality, a large head start may be infeasible due to concerns about fairness. In this case, our model predicts that hiring sabotage may exist in equilibrium because the head start is limited to an insufficient level. Similarly, head starts may incur additional psychological costs to new employees by lowering their morale, which can also reduce the optimal head start to a level that does not completely eliminate sabotage. Moreover, the principal may want a higher-ability agent to win the tournament, as she wants to select the strongest agent through the tournament and maximize *future* profits (Clark and Riis (2001); Hvide and Kristiansen (2003); Münster (2007); Ryvkin and Ortmann (2008); Brown and Minor (2014); Drugov and Ryvkin (2017)). In this case, providing a head start to the manager risks promoting a less competent agent, which harms the firm's future profitability. Consequently, the optimal head start may allow for some degree of hiring sabotage in equilibrium (i.e., $\theta_n^*(h^*) < \bar{\theta}$), but it will always ensure that the new hire is at least more capable than the manager (i.e., $\theta_n^*(h^*) > \theta_m$). This is consistent with Kawasaki's (2015) advice to have managers hire employees better than they are. We formalize this result in Appendix C.

Lastly, our model assumes that the manager has full discretion over hiring. In practice, hiring decisions may be made or overseen by committees involving senior managers or external evaluators, making managerial discretion incomplete. Nevertheless, our insights extend to such cases as long as the incumbent manager can influence the hiring decision. Moreover, while centralized hiring or oversight can mitigate hiring sabotage, it is desirable only when its cost is lower than the opportunity cost of delegating the decision to the manager.

A Proofs

A.1 Preliminaries

Lemma A.1. *The equilibrium effort levels e_m and e_n are bounded by $\bar{e} = \sup\{e : c(e)/e \leq 2\bar{\theta}\}$.*

Proof. First, a unique $\bar{e} = \sup\{e : c(e)/e \leq 2\bar{\theta}\} \in (0, \infty)$ exists by the assumptions on $c(e)$ (continuously differentiable, strictly increasing, strictly convex, and $\lim_{e \rightarrow \infty} c'(e) > 2\bar{\theta}$).

Now we prove the lemma by contradiction. Suppose WLOG that $e_m > \bar{e}$. Because m can always guarantee a nonnegative payoff by investing $e_m = 0$, we have $u_m = \alpha G(e_m - e_n + h)(e_m + e_n) - c(e_m)/\theta_m \geq 0$. Because $\alpha \leq 1$ and $G(e_m - e_n + h) \leq 1$, this implies

$$\frac{c(e_m)}{e_m} \leq \left(1 + \frac{e_n}{e_m}\right) \theta_m.$$

Since $e_m > \bar{e}$, we have

$$\frac{c(e_m)}{e_m} > 2\bar{\theta} \geq 2\theta_m.$$

Combining this with the previous inequality, we have $e_n > e_m$.

On the other hand, we also have $u_n \geq 0$ and thus

$$u_m + u_n = \alpha \cdot (e_m + e_n) - \frac{c(e_m)}{\theta_m} - \frac{c(e_n)}{\theta_n} \geq 0.$$

Because $c(e_m)/\theta_m > e_m$, we must have

$$\frac{c(e_n)}{\theta_n} \leq (e_m + e_n) - \frac{c(e_m)}{\theta_m} < e_n$$

and therefore $e_n \leq \bar{e} < e_m$, contradicting the previous conclusion that $e_n > e_m$. \square

Lemma A.2. *Assume $g(x)$ is twice continuously differentiable on its support, symmetric, and unimodal at $x = 0$. We have the following properties:*

- (i) $g'(x) \leq 0$ for all $x \geq 0$, and $g'(x) \geq 0$ for all $x \leq 0$.
- (ii) $J(x) \equiv \frac{G(x)}{g(x)} - x \geq \frac{1}{2g(0)}$ and is increasing (decreasing) when $x \geq 0$ ($x \leq 0$).
- (iii) For every $C < \infty$, as the variance of G grows arbitrarily large ($\sigma^2 \rightarrow \infty$), we have

$$\sup_{|x| \leq C} g(x) \rightarrow 0, \quad \sup_{|x| \leq C} g'(x) \rightarrow 0, \quad \sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)} \right| \rightarrow 0, \quad \sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)^2} \right| \rightarrow 0, \quad \sup_{|x| \leq C} |J'(x)| \rightarrow 0.$$

Proof. Part (i) is straightforward because $g(x)$ is single-peaked at zero.

To see (ii), by symmetry, we have $G(0) = 1/2$ and $J(0) = 1/2g(0)$. Taking the derivative yields

$$J'(x) = \frac{g(x)^2 - G(x)g'(x)}{g(x)^2} - 1 = -\frac{G(x)g'(x)}{g(x)^2},$$

which by (i), is positive (negative) if $x \geq 0$ ($x \leq 0$). Thus, $J(x)$ achieves its global minimum at $x = 0$.

To see (iii), by our parametric assumption, $G(x) = F(x/\sigma)$, where F is a standardized distribution with unit variance, and σ^2 is the variance of G . Then, we have $g(x) = f(x/\sigma)/\sigma$ and $g'(x) = f'(x/\sigma)/\sigma^2$. Because $f(x)$ is symmetric and unimodal at zero, $f'(0) = 0$.

To see (iii), by our parametric assumption, $G(x) = F(x/\sigma)$, where F is a standardized distribution with unit variance and density f . Then, we have $g(x) = f(x/\sigma)/\sigma$ and $g'(x) = f'(x/\sigma)/\sigma^2$.

Fix $C < \infty$. Since f is continuous and $f(0) > 0$, for all sufficiently large σ ,

$$\inf_{|x| \leq C} f(x/\sigma) \geq \frac{f(0)}{2}.$$

Moreover, since f is symmetric and differentiable at zero, $f'(0) = 0$. Since f' is continuously differentiable near zero,

$$\sup_{|x| \leq C} |f'(x/\sigma)| = \mathcal{O}(C/\sigma).$$

Therefore, we have the following conclusions as $\sigma \rightarrow \infty$.

$$\sup_{|x| \leq C} g(x) = \sup_{|x| \leq C} \frac{1}{\sigma} f(x/\sigma) = \mathcal{O}(1/\sigma) \rightarrow 0,$$

$$\sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)^2} \right| = \sup_{|x| \leq C} \left| \frac{f'(x/\sigma)}{f(x/\sigma)^2} \right| \leq \frac{\sup_{|x| \leq C} |f'(x/\sigma)|}{(\inf_{|x| \leq C} f(x/\sigma))^2} = \mathcal{O}(C/\sigma) \rightarrow 0,$$

and thus

$$\sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)} \right| \leq \sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)^2} \right| \cdot \sup_{|x| \leq C} g(x) \rightarrow 0,$$

$$\sup_{|x| \leq C} |g'(x)| \leq \sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)} \right| \cdot \sup_{|x| \leq C} g(x) \rightarrow 0.$$

Finally,

$$J'(x) = -\frac{G(x)g'(x)}{g(x)^2}.$$

Since $|G(x)| \leq 1$,

$$\sup_{|x| \leq C} |J'(x)| \leq \sup_{|x| \leq C} \left| \frac{g'(x)}{g(x)^2} \right| \rightarrow 0.$$

Therefore the convergence is uniform over every compact set $|x| \leq C$.

□

A.2 Proof of Lemma 1

Proof. Given θ_n , the first-order conditions are

$$\frac{\partial u_m}{\partial e_m} \equiv \alpha[(e_n + e_m) \cdot g(e_m - e_n + h) + G(e_m - e_n + h)] - c'(e_m)/\theta_m = 0, \quad (\text{A.1})$$

$$\frac{\partial u_n}{\partial e_n} \equiv \alpha[(e_n + e_m) \cdot g(e_m - e_n + h) + G(e_n - e_m - h)] - c'(e_n)/\theta_n = 0. \quad (\text{A.2})$$

When the variance is sufficiently large (or the cost is sufficiently convex), $g(x)$ and $|g'(x)/g(x)|$ are sufficiently small compared to $c''(e)$, so the second-order conditions are satisfied. Denote $x^* = e_m^* - e_n^* + h$, and

$$\begin{aligned} v_{mm} &\equiv \frac{\partial^2 u_m}{\partial e_m^2} = \alpha(2g + (e_m^* + e_n^*)g'(x^*)) - c''(e_m^*)/\theta_m < 0 \\ v_{nn} &\equiv \frac{\partial^2 u_n}{\partial e_n^2} = \alpha(2g - (e_m^* + e_n^*)g'(x^*)) - c''(e_n^*)/\theta_n < 0 \\ v_{mn} &\equiv \frac{\partial^2 u_m}{\partial e_m \partial e_n} = -\alpha(e_m^* + e_n^*)g'(x^*) \\ v_{nm} &\equiv \frac{\partial^2 u_n}{\partial e_n \partial e_m} = \alpha(e_m^* + e_n^*)g'(x^*) \end{aligned}$$

Denote the Jacobian matrix as

$$\mathbf{J}_1 = \begin{pmatrix} v_{mm} & v_{mn} \\ v_{nm} & v_{nn} \end{pmatrix}$$

and its determinant as

$$\det(\mathbf{J}_1) = v_{mm}v_{nn} - v_{mn}v_{nm} > 0.²¹$$

Denote $v_{mp} = \frac{\partial^2 u_m}{\partial e_m \partial p}$ and $v_{np} = \frac{\partial^2 u_n}{\partial e_n \partial p}$ for any parameter $p \in \{\theta_m, \theta_n, \alpha, h\}$. By the implicit function theorem,

$$\begin{pmatrix} de_m^*/dp \\ de_n^*/dp \end{pmatrix} = -\mathbf{J}_1^{-1} \begin{pmatrix} v_{mp} \\ v_{np} \end{pmatrix} \quad (\text{A.3})$$

²¹This implies the Nash equilibrium in the subgame is stable.

Therefore, (1) with respect to θ_m , because $v_{m,\theta_m} = c'(e_m^*)/\theta_m^2$ and $v_{n,\theta_m} = 0$, we have

$$\frac{de_m^*}{d\theta_m} = -\frac{c'(e_m^*)}{\theta_m^2} \frac{v_{nn}}{\det(\mathbf{J}_1)} > 0, \quad (\text{A.4})$$

$$\frac{de_n^*}{d\theta_m} = \frac{c'(e_m^*)}{\theta_m^2} \frac{\alpha(e_m^* + e_n^*)g'(x^*)}{\det(\mathbf{J}_1)} \stackrel{\text{sign}}{=} g'(x^*). \quad (\text{A.5})$$

By Lemma A.2, $g'(x) \leq 0$ ($g'(x) \geq 0$) if $x \geq 0$ ($x \leq 0$) because $g(x)$ is unimodal. Thus, $\frac{de_n^*}{d\theta_m} \geq 0$ ($\frac{de_n^*}{d\theta_m} \leq 0$) if n is more (less) likely to win. By symmetry, with respect to θ_n , we also have

$$\frac{de_n^*}{d\theta_n} = -\frac{c'(e_n^*)}{\theta_n^2} \frac{v_{mm}}{\det(\mathbf{J}_1)} > 0, \quad (\text{A.6})$$

$$\frac{de_m^*}{d\theta_n} = -\frac{c'(e_n^*)}{\theta_n^2} \frac{\alpha(e_m^* + e_n^*)g'(x^*)}{\det(\mathbf{J}_1)} \stackrel{\text{sign}}{=} -g'(x^*). \quad (\text{A.7})$$

As shown above, $\frac{de_m^*}{d\theta_n} \geq 0$ ($\frac{de_m^*}{d\theta_n} \leq 0$) if m is more (less) likely to win.

(2) With respect to α , because $v_{m\alpha} = (e_m^* + e_n^*)g(x^*) + G(x^*) = \frac{c'(e_m^*)}{\alpha\theta_m}$ and $v_{n\alpha} = (e_m^* + e_n^*)g(x^*) + G(-x^*) = \frac{c'(e_n^*)}{\alpha\theta_n}$, we have

$$\frac{de_m^*}{d\alpha} = \frac{-v_{nn}c'(e_m^*)/\alpha\theta_m - (e_m^* + e_n^*)g'(x^*)c'(e_n^*)/\theta_n}{\det(\mathbf{J}_1)} > 0, \quad (\text{A.8})$$

$$\frac{de_n^*}{d\alpha} = \frac{-v_{mm}c'(e_n^*)/\alpha\theta_n + (e_m^* + e_n^*)g'(x^*)c'(e_m^*)/\theta_m}{\det(\mathbf{J}_1)} > 0, \quad (\text{A.9})$$

(3) With respect to h , because $v_{mh} = \alpha((e_m^* + e_n^*)g'(x^*) + g(x^*))$ and $v_{nh} = \alpha((e_m^* + e_n^*)g'(x^*) - g(x^*))$, we have

$$\frac{de_m^*}{dh} = \alpha g(x^*) \frac{-2\alpha g(x^*) + \left(1 + (e_m^* + e_n^*)\frac{g'(x^*)}{g(x^*)}\right) c''(e_n^*)/\theta_n}{\det(\mathbf{J}_1)} > 0, \quad (\text{A.10})$$

$$\frac{de_n^*}{dh} = \alpha g(x^*) \frac{2\alpha g(x^*) - \left(1 - (e_m^* + e_n^*)\frac{g'(x^*)}{g(x^*)}\right) c''(e_m^*)/\theta_m}{\det(\mathbf{J}_1)} < 0, \quad (\text{A.11})$$

when the variance of G is sufficiently large (or when the cost is sufficiently convex).

(4) With respect to σ within a scale family $G(x) = F(x/\sigma)$, because

$$v_{m\sigma} = -\frac{\alpha}{\sigma}((e_m^* + e_n^*)(g(x^*) + x^*g'(x^*)) + x^*g(x^*)),$$

$$v_{n\sigma} = -\frac{\alpha}{\sigma}((e_m^* + e_n^*)(g(x^*) + x^*g'(x^*)) - x^*g(x^*)),$$

we have

$$\frac{de_m^*}{d\sigma} = -\frac{\alpha g(x^*)^2 \left((2e_m^* + h) + (e_m^* + e_n^*) x^* \frac{g'(x^*)}{g(x^*)} \right) c''(e_n^*)/\theta_n - 2\alpha g(x^*)(2e_m^* + h)}{\det(\mathbf{J}_1)}, \quad (\text{A.12})$$

$$\frac{de_n^*}{d\sigma} = -\frac{\alpha g(x^*)^2 \left((2e_n^* - h) + (e_m^* + e_n^*) x^* \frac{g'(x^*)}{g(x^*)} \right) c''(e_m^*)/\theta_m - 2\alpha g(x^*)(2e_n^* - h)}{\det(\mathbf{J}_1)}, \quad (\text{A.13})$$

For sufficiently large variance of G (or sufficiently convex cost), $\frac{de_m^*}{d\sigma} < 0$ if $2e_m^* + h > 0$, and $\frac{de_n^*}{d\sigma} < 0$ if $2e_n^* - h > 0$. Sufficient conditions are $h \geq 0$ and $h \leq 0$, respectively.

(5) We show that if $\theta_m \geq \theta_n$ ($\theta_m \leq \theta_n$) and $h \geq 0$ ($h \leq 0$), then $x^* \geq 0$ ($x^* \leq 0$), i.e., the manager is more (less) likely to win.

First, note that when $h = 0$ and $\theta_m = \theta_n$, we have $e_m^* = e_n^*$ and thus $x^* = 0$.

Then, because

$$\frac{dx^*}{d\theta_m} = \frac{de_m^*}{d\theta_m} - \frac{de_n^*}{d\theta_m} = \frac{c'(e_m^*) c''(e_n^*)/\theta_n - 2\alpha g(x^*)}{\theta_m^2 \det(\mathbf{J}_1)} > 0, \quad (\text{A.14})$$

$\frac{dx^*}{d\theta_n} < 0$ (by symmetry), and

$$\frac{dx^*}{dh} = \frac{de_m^*}{dh} - \frac{de_n^*}{dh} + 1 > 0, \quad (\text{A.15})$$

we have $x^* \geq 0$ if $\theta_m \geq \theta_n$ and $h \geq 0$. By symmetry, $x^* \leq 0$ if $\theta_n \geq \theta_m$ and $h \leq 0$.

(6) Finally, because efforts are bounded, there exists $\bar{h}_1, \bar{h}_2 > 0$ such that have $x^* = e_m^* - e_n^* + h \geq 0$ for all $h > \bar{h}_1$ and $x^* = e_m^* - e_n^* + h \leq 0$ for all $h < -\bar{h}_2$. \square

A.3 Proof of Lemma 2

Proof. Denote $x(\theta_n) = e_m(\theta_n) - e_n(\theta_n) + h$ and $J(x) = G(x)/g(x) - x$. Then, for all $\theta_n \in \Theta$, we have

$$\frac{du_m(e_m(\theta_n), e_n(\theta_n), \theta_n)}{d\theta_n} = \alpha e_n'(\theta_n) [J(x(\theta_n)) - (2e_n(\theta_n) - h)] g(x(\theta_n)). \quad (\text{A.16})$$

First, we show that u_m is quasiconcave in θ_n . Define $z(\theta_n) = J(x(\theta_n)) - (2e_n(\theta_n) - h)$. We have

$$\begin{aligned} z'(\theta_n) &= J'(x(\theta_n)) x'(\theta_n) - 2e_n'(\theta_n) \\ &= \frac{c'(e_n)}{\theta_n^2 \det(\mathbf{J}_1)} \left(2\alpha [(e_m + e_n) g'(x) + (J'(x) + 2) g(x)] - (J'(x) + 2) \frac{c''(e_m)}{\theta_m} \right) \end{aligned}$$

where $e_m = e_m(\theta_n)$, $e_n = e_n(\theta_n)$, and $x = x(\theta_n)$.

To determine the sign of $z'(\theta_n)$ when σ is sufficiently large, first note that, by Lemma A.1, $e_m(\theta_n)$ and $e_n(\theta_n)$ are bounded by \bar{e} for all $\theta_n \in \Theta$. Because $h \leq \bar{H}$, we have $x(\theta_n) = e_m(\theta_n) - e_n(\theta_n) + h \leq 2\bar{e} + \bar{H}$ for all $\theta_n \in \Theta$. Therefore, by Lemma A.2, we have

$$\sup_{\theta_n \in \Theta} |g(x(\theta_n))| \rightarrow 0, \quad \sup_{\theta_n \in \Theta} |g'(x(\theta_n))| \rightarrow 0, \quad \sup_{\theta_n \in \Theta} |J'(x(\theta_n))| \rightarrow 0$$

as $\sigma \rightarrow \infty$. Now we decompose the bracketed term into two parts:

$$\begin{aligned} B_1(\theta_n) &= 2\alpha [(e_m(\theta_n) + e_n(\theta_n))g'(x(\theta_n)) + (J'(x(\theta_n)) + 2)g(x(\theta_n))] \\ B_2(\theta_n) &= -(J'(x(\theta_n)) + 2) \frac{c''(e_m(\theta_n))}{\theta_m}. \end{aligned}$$

As $\sigma \rightarrow \infty$, we have $\sup_{\theta_n \in \Theta} |B_1(\theta_n)| \rightarrow 0$ and

$$\sup_{\theta_n \in \Theta} \left| B_2(\theta_n) + 2 \frac{c''(e_m(\theta_n))}{\theta_m} \right| \rightarrow 0.$$

Because $c''(e_m(\theta_n)) > 0$, there exists $\bar{\sigma} < \infty$ such that, for all $\sigma \geq \bar{\sigma}$,

$$B_1(\theta_n) + B_2(\theta_n) < 0 \quad \text{for all } \theta_n \in \Theta.$$

Thus, we have $z'(\theta_n) < 0$ for all $\theta_n \in \Theta$ when the variance is sufficiently large.²²

Hence, u_m is quasiconcave in θ_n , and there exists a unique $\theta_n^* \in \Theta$ (possibly at the boundary) that satisfies $u'_m(\theta_n) \geq 0$ if and only if $\theta_n \leq \theta_n^*$. In this lemma, we focus on the interior solution $\theta_n^* \in (\underline{\theta}, \bar{\theta})$ such that $z(\theta_n^*) = 0$, $u'_m(\theta_n^*) = 0$, and $2e_n(\theta_n^*) - h = J(x(\theta_n^*)) > 0$.

When $\theta_n^* \in (\underline{\theta}, \bar{\theta})$, the first-order conditions are

$$c'(e_n^*)/\theta_n^* = \alpha, \tag{A.17}$$

$$c'(e_m^*)/\theta_m = 2\alpha \cdot G(e_m^* - e_n^* + h), \tag{A.18}$$

$$G(e_m^* - e_n^* + h) = (e_m^* + e_n^*) \cdot g(e_m^* - e_n^* + h). \tag{A.19}$$

Denote $e_m^* = e_m(\theta_n^*)$, $e_n^* = e_n(\theta_n^*)$, and $x^* = x(\theta_n^*)$. Denote $w_m = c'(e_m^*)/\theta_m - 2\alpha \cdot G(e_m^* - e_n^* + h)$ and $w_n = G(e_m^* - e_n^* + h) - (e_m^* + e_n^*) \cdot g(e_m^* - e_n^* + h)$. Define w_m and w_n analogously to v_m and v_n in the proof of Lemma 1.

²²Instead of assuming a sufficiently large variance, another common assumption in the literature is that the cost function is sufficiently convex. When the inverse Mills ratio $g(x)/G(x)$ is decreasing (i.e., G is log-concave), a sufficiently convex cost function also implies $z'(\theta_n) < 0$. To see this, when G is log-concave, we have $J'(x) \geq -1$, which implies $-(J'(x) + 2) \frac{c''(e_m)}{\theta_m}$ is negative and becomes arbitrarily large when the cost is sufficiently convex. Since the remaining terms are uniformly bounded, we have $z'(\theta_n) < 0$ for all $\theta_n \in \Theta$ when $c''(e_m)$ is sufficiently large.

Denote the Jacobian matrix as

$$\mathbf{J}_2 = \begin{pmatrix} w_{mm} & w_{mn} \\ w_{nm} & w_{nn} \end{pmatrix} = \begin{pmatrix} c''(e_m^*)/\theta_m - 2\alpha g(x^*) & 2\alpha g(x^*) \\ -(e_m^* + e_n^*)g'(x^*) & -2g(x^*) + (e_m^* + e_n^*)g'(x^*) \end{pmatrix}.$$

By the implicit function theorem, for any parameter $p \in \{\theta_m, \alpha, h\}$,

$$\begin{pmatrix} de_m^*/dp \\ de_n^*/dp \end{pmatrix} = -\mathbf{J}_2^{-1} \begin{pmatrix} w_{mp} \\ w_{np} \end{pmatrix} \quad (\text{A.20})$$

and

$$\frac{d\theta_n^*}{dp} = \frac{c''(e_n^*)}{\alpha} \frac{de_n^*}{dp} - \frac{c'(e_n^*)}{\alpha^2} \frac{d\alpha}{dp}.$$

By Lemma A.2, when the variance is sufficiently large, we have

$$A \equiv w_{mm} = \frac{c''(e_m^*)}{\theta_m} - 2\alpha g(x^*) > 0, \quad (\text{A.21})$$

$$B \equiv -\frac{w_{nn}}{g(x^*)} = 2 - (e_m^* + e_n^*) \frac{g'(x^*)}{g(x^*)} > 0, \quad (\text{A.22})$$

$$D \equiv -\frac{\det(\mathbf{J}_2)}{g(x^*)} = \frac{c''(e_m^*)}{\theta_m} B - 4\alpha g(x^*) > 0, \quad (\text{A.23})$$

$$D - A = \frac{c''(e_m^*)}{\theta_m} (B - 1) - 2\alpha g(x^*) > 0 \quad (\text{A.24})$$

and thus second-order conditions are satisfied (alternatively, they hold when the cost is sufficiently convex).

(1) With respect to θ_m , because $w_{m,\theta_m} = -c'(e_m^*)/\theta_m^2$ and $w_{n,\theta_m} = 0$, we have

$$\frac{de_m^*}{d\theta_m} = \frac{B}{D} \frac{c'(e_m)}{\theta_m^2} > 0, \quad (\text{A.25})$$

$$\frac{de_n^*}{d\theta_m} = -\frac{1}{D} (e_m^* + e_n^*) \frac{c'(e_m)}{\theta_m^2} \frac{g'(x^*)}{g(x^*)} \stackrel{\text{sign}}{=} -g'(x^*), \quad (\text{A.26})$$

$$\frac{d\theta_n^*}{d\theta_m} = \frac{c''(e_n^*)}{\alpha} \frac{de_n^*}{d\theta_m} \stackrel{\text{sign}}{=} -g'(x^*). \quad (\text{A.27})$$

By Lemma A.2, $g'(x) \leq 0$ ($g'(x) \geq 0$) if $x \geq 0$ ($x \leq 0$) because $g(x)$ is single-peaked at zero.

Thus, $\frac{de_n^*}{d\theta_m}$ and $\frac{d\theta_n^*}{d\theta_m}$ are positive (negative) if m is more (less) likely to win.

(2) With respect to α , because $w_{m\alpha} = -2G(x^*) = -c'(e_m^*)/\alpha\theta_m$ and $w_{n\alpha} = 0$, we have

$$\frac{de_m^*}{d\alpha} = \frac{B}{D} \frac{c'(e_m)}{\alpha\theta_m} > 0, \quad (\text{A.28})$$

$$\frac{de_n^*}{d\alpha} = -\frac{1}{D} \frac{c'(e_m)}{\alpha\theta_m} (e_m^* + e_n^*) \frac{g'(x^*)}{g(x^*)} \stackrel{\text{sign}}{=} -g'(x^*), \quad (\text{A.29})$$

$$\frac{d\theta_n^*}{d\alpha} = \frac{c''(e_n^*)}{\alpha} \frac{de_n^*}{d\alpha} - \frac{c'(e_n^*)}{\alpha^2} < 0. \quad (\text{A.30})$$

The last equation is negative if $g'(x^*) \geq 0$, because then $de_n^*/d\alpha \leq 0$. When $g'(x^*) \leq 0$, it holds under the large-variance condition, since $|g'(x^*)/g(x^*)|$ is sufficiently small and hence $de_n^*/d\alpha$ is sufficiently small.

(3) With respect to h , because $w_{mh} = -2\alpha g(x^*)$ and $w_{nh} = g(x^*) - (e_m^* + e_n^*)g'(x^*)$, we have

$$\frac{de_m^*}{dh} = \frac{2\alpha g(x^*)}{D} > 0, \quad (\text{A.31})$$

$$\frac{de_n^*}{dh} = \frac{D - A}{D} > 0, \quad (\text{A.32})$$

$$\frac{d\theta_n^*}{dh} = \frac{c''(e_n^*)}{\alpha} \frac{de_n^*}{dh} > 0. \quad (\text{A.33})$$

Moreover, holding the new hire's ability fixed, we have

$$\frac{\partial e_n(\theta_n^*, h)}{\partial h} = -\alpha g(x^*) \frac{D - A}{\det(\mathbf{J}_1)} < 0. \quad (\text{A.34})$$

Thus, the discouragement effect is negative but is dominated by the hiring effect.

(4) With respect to σ and within a scale family $G(x) = F(x/\sigma)$, because $w_{m\sigma} = 2\alpha x^* g(x^*)/\sigma$ and $w_{n\sigma} = ((e_m^* + e_n^*)(g(x^*) + x^* g'(x^*)) - x^* g(x^*))/\sigma$,

$$\frac{de_m^*}{d\sigma} = -\frac{2\alpha(2e_m^* + h)g(x^*)}{\sigma D} < 0 \quad \text{if } h \geq 0, \quad (\text{A.35})$$

$$\frac{de_n^*}{d\sigma} = \frac{A \cdot J(x^*) + \frac{c''(e_m^*)}{\theta_m} (e_m^* + e_n^*) x^* \frac{g'(x^*)}{g(x^*)}}{\sigma D} > 0, \quad (\text{A.36})$$

$$\frac{d\theta_n^*}{d\sigma} = \frac{c''(e_n^*)}{\alpha} \frac{de_n^*}{d\sigma} > 0 \quad (\text{A.37})$$

The second inequality follows from $A > 0$ and $J(x^*) \geq 1/(2g(0)) > 0$. The remaining term is dominated under the large-variance condition because $|g'(x^*)/g(x^*)|$ is sufficiently small.

(5) We show that there exists $\bar{\theta}_m, \underline{\theta}_m \in \Theta$ such that if $\theta_m > \bar{\theta}_m$ ($\theta_m < \underline{\theta}_m$) and $h \geq 0$ ($h \leq 0$), then $x^* \geq 0$ ($x^* \leq 0$), i.e., the manager is more (less) likely to win.

First, note that when $h = 0$ and $\theta_m \rightarrow 0$, we have $e_m^* \rightarrow 0$, so the first-order condition (A.16)

implies

$$G(-e_n^*) = e_n^* g(-e_n^*) \iff e_n^* = \frac{1 - G(e_n^*)}{g(e_n^*)} > 0. \quad (\text{A.38})$$

Therefore, $x^* = -e_n^* < 0$. On the other hand, when $h = 0$ and $\theta_m = \bar{\theta}$, we have $\theta_n^* \leq \theta_m$. Therefore, by Lemma 1, $x^* \geq 0$.

Then, because

$$\frac{dx^*}{d\theta_m} = \frac{de_m^*}{d\theta_m} - \frac{de_n^*}{d\theta_m} = \frac{c'(e_m^*)}{\theta_m^2} \frac{2}{D} > 0, \quad (\text{A.39})$$

and

$$\frac{dx^*}{dh} = \frac{de_m^*}{dh} - \frac{de_n^*}{dh} + 1 = \frac{2\alpha g(x^*) + A}{D} > 0, \quad (\text{A.40})$$

we have $x^* \geq 0$ if $\theta_m \geq \bar{\theta}_m$ and $h \geq 0$. Similarly, we also have $x^* \leq 0$ if $\theta_m \leq \underline{\theta}_m$ and $h \leq 0$.

(6) Finally, because equilibrium efforts are bounded, there exists $\bar{h}_1, \bar{h}_2 > 0$ such that have $x^* = e_m^* - e_n^* + h \geq 0$ for all $h > \bar{h}_1$ and $x^* = e_m^* - e_n^* + h \leq 0$ for all $h < -\bar{h}_2$. \square

A.4 Proof of Lemma 3

Proof. Given θ_n , the first-order conditions are

$$u_m \equiv \alpha[(e_n + e_m) \cdot g(e_m - e_n + h) + G(e_m - e_n + h)] - c'(e_m)/\theta_m = 0, \quad (\text{A.41})$$

$$v_n \equiv \alpha[(e_n + e_m) \cdot g(e_m - e_n + h) + G(e_n - e_m - h)] - c'(e_n)/\theta_n = 0. \quad (\text{A.42})$$

We have

$$\frac{de_m^*}{dh} + \frac{de_n^*}{dh} = \alpha \frac{(g(x^*) + (e_m^* + e_n^*)g'(x^*)) \frac{c''(e_n^*)}{\theta_n} - (g(x^*) - (e_m^* + e_n^*)g'(x^*)) \frac{c''(e_m^*)}{\theta_m}}{\det(\mathbf{J}_1)}. \quad (\text{A.43})$$

Thus, $\frac{de_m^*}{dh} + \frac{de_n^*}{dh} \leq 0$ if and only if

$$(e_m^* + e_n^*) \frac{g'(x^*)}{g(x^*)} \leq \frac{c''(e_m^*)/\theta_m - c''(e_n^*)/\theta_n}{c''(e_m^*)/\theta_m + c''(e_n^*)/\theta_n}. \quad (\text{A.44})$$

With quadratic costs $c(e) = e^2/2$, this condition simplifies to

$$(e_m^* + e_n^*) \frac{g'(e_m^* - e_n^* + h)}{g(e_m^* - e_n^* + h)} \leq \frac{\theta_n - \theta_m}{\theta_n + \theta_m}, \quad (\text{A.45})$$

which is satisfied if $\theta_n > \theta_m$ and the variance is sufficiently large because $|g'(x)/g(x)|$ is small. \square

A.5 Proofs of Proposition 2 and Corollary

Proof of Proposition 2. Solving $\max_{\alpha}(1 - \alpha)(\alpha\bar{\theta} + \frac{2\alpha\bar{\theta}}{M/\alpha\theta_m - 2})$ gives

$$\alpha^* = \frac{M - \sqrt{M(M - 2\theta_m)}}{2\theta_m} = \left(1 + \sqrt{1 - 2\frac{\theta_m}{M}}\right)^{-1}.$$

The principal's profit is

$$\Pi^* = (1 - \alpha^*)(\alpha^*\bar{\theta} + \frac{2\alpha^*\bar{\theta}}{M/\alpha\theta_m - 2}) = \frac{M(M - \theta_m - \sqrt{M(M - 2\theta_m)})}{2\theta_m^2}\bar{\theta} = \left(2\left(\frac{1}{\alpha^*} - \frac{\theta_m}{M}\right)\right)^{-1}\bar{\theta}.$$

□

Proof of Corollary 2.1. For $\alpha^* = \frac{M - \sqrt{M(M - 2\theta_m)}}{2\theta_m}$, because $M > 2\bar{\theta} \geq 2\theta_m$,

$$\frac{d\alpha^*}{dM} = \frac{\sqrt{M(M - 2\theta_m)} - (M - \theta_m)}{2\theta_m\sqrt{M(M - 2\theta_m)}} < 0. \quad (\text{A.46})$$

$$\frac{d\alpha^*}{d\theta_m} = -\frac{\sqrt{M(M - 2\theta_m)} - (M - \theta_m)}{2\theta_m^2\sqrt{M(M - 2\theta_m)}}M > 0. \quad (\text{A.47})$$

For $h^* = 2\alpha^*\bar{\theta} - M/2$,

$$\frac{dh^*}{dM} = 2\frac{d\alpha^*}{dM}\bar{\theta} - \frac{1}{2} < 0. \quad (\text{A.48})$$

$$\frac{dh^*}{d\theta_m} = 2\frac{d\alpha^*}{d\theta_m}\bar{\theta} > 0. \quad (\text{A.49})$$

Because $\Pi^* = \max_{\alpha}(1 - \alpha)(\alpha\bar{\theta} + \frac{2\alpha\bar{\theta}}{M/\alpha\theta_m - 2})$, by the envelope theorem, we have $\frac{d\Pi^*}{dM} < 0$ and $\frac{d\Pi^*}{d\theta_m} > 0$. □

B General Prize Sharing Rules

We consider a general sharing rule where the winner receives share α and the loser receives share β , where $0 \leq \beta \leq \alpha \leq 1$, instead of a winner-take-all tournament. Let $\gamma = \alpha - \beta \in [0, \alpha]$ denote the spread and $\tau = \alpha + \beta \in [0, 1]$ denote the total share. In particular, the winner-take-all tournament corresponds to $\beta = 0$ and hence $\gamma = \tau = \alpha$. The principal's profit is given by $\Pi(\alpha, \beta) = (1 - \tau)(e_m^* + e_n^*)$.

Given the new hire's ability θ_n , in the tournament stage, the expected payoffs for m and n are

given by

$$u_m(e_m, e_n, \theta_m) = [\beta + \gamma G(e_m - e_n + h)](e_m + e_n) - \frac{c(e_m)}{\theta_m}, \quad (\text{B.1})$$

$$u_n(e_m, e_n, \theta_n) = [\beta + \gamma G(e_n - e_m - h)](e_m + e_n) - \frac{c(e_n)}{\theta_n}. \quad (\text{B.2})$$

The first-order conditions for the tournament stage are

$$\frac{\partial u_m}{\partial e_m} = \gamma \cdot (e_m + e_n)g(e_m - e_n + h) + \beta + \gamma G(e_m - e_n + h) - \frac{c'(e_m)}{\theta_m} = 0, \quad (\text{B.3})$$

$$\frac{\partial u_n}{\partial e_n} = \gamma \cdot (e_m + e_n)g(e_m - e_n + h) + \beta + \gamma G(e_n - e_m - h) - \frac{c'(e_n)}{\theta_n} = 0. \quad (\text{B.4})$$

The equilibrium effort levels $e_m(\theta_n)$ and $e_n(\theta_n)$ in the tournament stage are implicitly defined by the above first-order conditions.

Running Example (Uniform–Quadratic). Suppose $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, with M sufficiently large so that all winning probabilities are interior. Then,

$$e_m(\theta_n) = \frac{\tau M/2 + \gamma h}{M/\theta_m - 2\gamma},$$

$$e_n(\theta_n) = \frac{\tau M/2 - \gamma h}{M/\theta_n - 2\gamma}.$$

Fixing the spread γ , a larger total share τ increases both the manager's and the new hire's efforts. On the other hand, fixing the total share τ , a wider spread γ increases the manager's effort if and only if $h \geq -\tau\theta_m$ and increases the new hire's effort if and only if $h \leq \tau\theta_n$.

Then, at the hiring stage, the manager chooses θ_n to maximize her expected payoff in the tournament stage, and the first-order condition is

$$\frac{du_m}{d\theta_n} = \frac{\partial u_m}{\partial e_n} e'_n(\theta_n) = [\beta + \gamma G(e_m - e_n + h) - \gamma \cdot (e_m + e_n)g(e_m - e_n + h)] e'_n(\theta_n). \quad (\text{B.5})$$

Hence, in the SPNE, when the hiring decision $\theta_n^* \in (\underline{\theta}, \bar{\theta})$, the first-order conditions of θ_n^* and the equilibrium effort levels (e_m^*, e_n^*) are given by

$$\beta + \gamma G(e_m^* - e_n^* + h) = \gamma \cdot (e_m^* + e_n^*)g(e_m^* - e_n^* + h), \quad (\text{B.6})$$

$$c'(e_n^*)/\theta_n^* = \tau, \quad (\text{B.7})$$

$$c'(e_m^*)/\theta_m = 2[\beta + \gamma G(e_m^* - e_n^* + h)]. \quad (\text{B.8})$$

In general, when the prize spread $\gamma > 0$ (i.e., $\alpha > \beta$), the analysis is similar to the benchmark

model. First, similar to Lemmas 2 and 3, the head start has a positive encouragement effect on the manager (i.e., $de_m^*/dh > 0$), and its hiring effect dominates the discouragement effect (i.e., $de_n^*/dh > 0$) until the strongest candidate is hired (i.e., $\theta_n^* < \bar{\theta}$). Then, once the strongest candidate is hired, under condition (9), any further head start leads the discouragement effect to dominate the encouragement effect, so the expected total output decreases in h . Therefore, Proposition 1 continues to hold: for any given $\alpha, \beta \in (0, 1)$ with $\alpha > \beta$, the optimal head start is $h^* = \bar{h}(\alpha, \beta)$, the smallest head start that induces the manager to hire the strongest candidate.

When the prize spread $\gamma = 0$ (i.e., $\alpha = \beta$), the manager has no incentive to sabotage hiring, so she always hires the strongest candidate $\theta_n^* = \bar{\theta}$, and the equilibrium effort levels are given by $c'(e_m^*) = \alpha\theta_m$ and $c'(e_n^*) = \alpha\bar{\theta}$. Thus, the head start has no effect on total effort, and it is without loss to define $\bar{h}(\alpha, \beta) = 0$ when $\alpha = \beta$.

Running Example (Uniform–Quadratic). For $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, the equilibrium efforts and the optimal hiring decision are given by

$$\begin{aligned}\theta_n^* &= \min \left\{ \frac{\tau M/2 + \gamma h}{2\gamma\tau}, \bar{\theta} \right\}, \\ e_m^* &= \frac{\tau M/2 + \gamma h}{M/\theta_m - 2\gamma}, \\ e_n^* &= \min \left\{ \frac{\tau M/2 + \gamma h}{2\gamma}, \frac{\tau M/2 - \gamma h}{M/\bar{\theta} - 2\gamma} \right\}.\end{aligned}$$

If $\gamma > 0$, the smallest head start that induces the manager to hire the strongest candidate is given by $\bar{h}(\alpha, \beta) = \left(2\bar{\theta} - \frac{M}{2\gamma}\right)\tau$. In the corner case $\gamma = 0$ (i.e., $\alpha = \beta = \tau/2$), we have $e_m^* = \alpha\theta_m = \tau\theta_m/2$ and $e_n^* = \alpha\bar{\theta} = \tau\bar{\theta}/2$.

Fixing the total share τ , a wider spread γ increases the manager's effort if and only if $h \geq -\tau\theta_m$. It also increases her incentive to sabotage, but the aggregate effect on the new hire's effort is negative until the strongest candidate is hired (i.e., if $\theta_n^* < \bar{\theta}$). On the other hand, fixing the spread, a higher total share τ always increases the manager's effort. It also increases her incentive to sabotage if and only if $h \geq 0$, and the aggregate effect on the new hire's effort is always positive until the strongest candidate is hired (i.e., if $\theta_n^* < \bar{\theta}$).

Finally, we show that the optimal sharing rule is winner-take-all (i.e., $\beta^* = 0$) in the uniform–quadratic setting.

Proposition B.1. *Suppose $G \sim \text{Unif}[-\frac{M}{2}, \frac{M}{2}]$ and $c(e) = e^2/2$, with $M > 2\bar{\theta}$ sufficiently large. Then, the profit-maximizing principal will choose a winner-take-all tournament (i.e., $\beta^* = 0$ and $\gamma^* = \alpha^*$) with head start $h^* = \bar{h}(\alpha^*, 0)$ that induces the manager to hire the strongest candidate.*

Proof. First, we show that $\gamma = 0$ (i.e., $\alpha = \beta = \tau/2$) cannot be optimal. In this case, the manager

has no incentive to sabotage hiring (i.e., $\theta_n^* = \bar{\theta}$). With $c(e) = e^2/2$, equilibrium effort levels are given by $e_m^\circ = \alpha\theta_m = \tau\theta_m/2$ and $e_n^\circ = \alpha\bar{\theta} = \tau\bar{\theta}/2$. Therefore, fixing the total share τ , the expected total output is given by

$$e_m^\circ + e_n^\circ = \frac{\theta_m + \bar{\theta}}{2}\tau.$$

In the uniform–quadratic setting, the optimal head start, which induces the manager to hire the strongest candidate, is given by $h^* = \bar{h}(\alpha, \beta) = \left(2\bar{\theta} - \frac{M}{2\gamma}\right)\tau$. For any $\gamma > 0$, fixing the total share τ , the expected total output when $h^* = \bar{h}(\alpha, \beta)$ is given by

$$e_m^*(h^*(\gamma), \gamma) + e_n^*(h^*(\gamma), \gamma) = \frac{M\bar{\theta}}{M - 2\gamma\theta_m}\tau > \bar{\theta}\tau > e_m^\circ + e_n^\circ.$$

Therefore, $\gamma = 0$ cannot be optimal.

Then, we show that if $\gamma > 0$, fixing the total share τ , the expected total output $e_m^*(h^*(\gamma), \gamma) + e_n^*(h^*(\gamma), \gamma)$ is increasing in the spread γ . In the uniform–quadratic setting, we have

$$\frac{d}{d\gamma}[e_m^*(h^*(\gamma), \gamma) + e_n^*(h^*(\gamma), \gamma)] = \frac{2\tau\bar{\theta}M\theta_m}{(M - 2\gamma\theta_m)^2} > 0 \quad (\text{B.9})$$

when $M > 2\bar{\theta} \geq 2\gamma\theta_m$ and $\gamma > 0$. Thus, the expected total output is increasing in the spread γ .

Therefore, to maximize profit $\Pi(\alpha, \beta) = (1 - \tau)(e_m^* + e_n^*)$, the principal chooses the largest possible spread $\gamma^* = \alpha^* = \tau^*$, or equivalently, a winner-take-all tournament where $\beta^* = 0$. \square

Since the optimal sharing rule is winner-take-all (i.e., $\beta^* = 0$), the optimal head start h^* and payout ratio α^* characterized in Proposition 2 remain unchanged under the general sharing rule. In summary, our results show that giving a head start h to the manager is a more effective instrument for mitigating hiring sabotage than giving a positive share β to the loser: when the principal can use a head start, the optimal tournament is winner-take-all.

C A Two-Period Model

C.1 Setup

In the main text, we show that the optimal head start always ensures that the manager hires the strongest candidate—that is, it eliminates hiring sabotage. To examine the robustness of this result, we now extend the model to two periods to account for future profitability of the firm by assuming that the tournament winner is promoted (or retained) in the next period. As common in practice, the firm uses the same rule for both compensation and promotion for the sake of simplicity and

transparency. Thus, the principal now incorporates the winner's ability into her objectives, as promoting a higher-ability agent from the tournament is vital for future profitability.

Assume the promoted agent receives a constant continuation payoff $\tilde{v} > 0$. After the first-period tournament, the principal receives a continuation payoff $\tilde{V}(\theta) > 0$ if the agent who is promoted (i.e., who wins in the first period) has ability θ . Assume that $\tilde{V}(\theta)$ is increasing.

Alternatively, the continuation payoff \tilde{V} can be interpreted as capturing the cost of providing head starts in the one-period model, which is incurred when a weaker player wins the tournament. When a weaker player wins the tournament, the principal's reputation may suffer or its future profit may decline.

C.2 Agent's Problem

For simplicity, we focus on the uniform-quadratic case where $c(e) = e^2/2$ and $G \sim \text{Unif}[-M/2, M/2]$. Denote by $\delta \in (0, 1)$ the discount factor. Given the manager's hiring decision, θ_n , the expected payoff of m and n in the subgame are given by

$$u_m = \alpha \cdot (e_m + e_n)G(e_m - e_n + h) - \frac{e_m^2}{2\theta_m} + \delta G(e_m - e_n + h) \cdot \tilde{v}, \quad (\text{C.1})$$

$$u_n = \alpha \cdot (e_m + e_n)G(e_n - e_m - h) - \frac{e_n^2}{2\theta_n} + \delta G(e_n - e_m - h) \cdot \tilde{v}. \quad (\text{C.2})$$

Analogous to the one-period case, the equilibrium efforts in the tournament stage given θ_n are

$$e_m(\theta_n) = \frac{(M/2 + h) + \delta\tilde{v}/\alpha}{M/\alpha\theta_m - 2}, \quad (\text{C.3})$$

$$e_n(\theta_n) = \frac{(M/2 - h) + \delta\tilde{v}/\alpha}{M/\alpha\theta_n - 2}. \quad (\text{C.4})$$

At the hiring stage in the first period, the manager chooses θ_n^* according to the first-order condition

$$\frac{du_m}{d\theta_n} = e'_n(\theta_n)[\alpha \cdot (G(e_m - e_n + h) - g(e_m - e_n + h)(e_m + e_n)) - \delta g(e_m - e_n + h) \cdot \tilde{v}] = 0. \quad (\text{C.5})$$

Therefore, in the SPNE,

$$\theta_n^* = \min \left(\frac{(M/2 + h) - \delta\tilde{v}/\alpha}{2\alpha}, \bar{\theta} \right), \quad (\text{C.6})$$

$$e_n^* \equiv e_n(\theta_n^*) = \min \left(\frac{(M/2 + h) - \delta\tilde{v}/\alpha}{2}, \frac{(M/2 - h) + \delta\tilde{v}/\alpha}{M/\alpha\bar{\theta} - 2} \right), \quad (\text{C.7})$$

$$e_m^* = \frac{(M/2 + h) + \delta\tilde{v}/\alpha}{M/\alpha\theta_m - 2}. \quad (\text{C.8})$$

The amount of head start necessary to induce the manager to hire the strongest candidate is

$$\bar{h}(\alpha) = 2\alpha\bar{\theta} - \frac{M}{2} + \delta\tilde{v}/\alpha > 2\alpha\bar{\theta} - \frac{M}{2}, \quad (\text{C.9})$$

which is larger than that in the one-period model where $\bar{h}(\alpha) = 2\alpha\bar{\theta} - M/2$ because of the continuation value. We assume that M is sufficiently large so that the winning probabilities lies in $(0, 1)$ when the head start is $\bar{h}(\alpha)$.

As we shall see below, the *encouragement*, *discouragement*, and *hiring effects* observed in the one-period model remain for the agent, and Lemma 1 still holds.

C.3 Principal's Problem

The principal's problem is

$$\begin{aligned} \max_{\alpha \in [0,1], h \in [-\bar{H}, \bar{H}]} \tilde{\Pi}(\alpha, h) &= (1 - \alpha)(e_m^* + e_n^*) + \delta\tilde{V}(\theta_m)G(e_m^* - e_n^* + h) \\ &+ \delta\tilde{V}(\theta_n^*(h))[1 - G(e_m^* - e_n^* + h)]. \end{aligned} \quad (\text{C.10})$$

Using two-step maximization again, the first-order condition for h is

$$\begin{aligned} \frac{d\tilde{\Pi}}{dh} &= \underbrace{(1 - \alpha)\left(\frac{de_n}{dh} + \frac{de_m}{dh}\right)}_{\text{first-period effects } (> 0 \text{ if } \theta_n^* < \bar{\theta})} + \underbrace{\delta(\tilde{V}(\theta_m) - \tilde{V}(\theta_n^*(h)))\left(\frac{de_m}{dh} - \frac{de_n}{dh} + 1\right)g(e_m^* - e_n^* + h)}_{\text{Succession effect}} \\ &\quad + \underbrace{\delta\tilde{V}'(\theta_n^*(h))[1 - G(e_m^* - e_n^* + h)]\theta_n^{*'}(h)}_{\text{Extended hiring effect}}. \end{aligned} \quad (\text{C.11})$$

For simplicity, assume that the principal's continuation payoff is given by $\tilde{V}(\theta) = k\theta + b$ with $k, b > 0$, which captures the succession concern.

Succession and Hiring Effects. Until the strongest candidate is hired (when $\theta_n^* < \bar{\theta}$), the head start increases the manager's effort because of the *encouragement effect* ($de_m^*/dh > 0$) and also increases the new hire's effort as the *hiring effect* still dominates the *discouragement effect* ($de_n^*/dh = 1/2$ as in equation (3)). Thus, they aggregate to a positive effect on the profit as long as $\theta_n^* < \bar{\theta}$ as in Lemma 1. However, in a model with succession concerns, a head start has two additional effects:

4. Extended hiring effect: $[1 - G(e_m - e_n + h)]\delta k\theta_n^{*'}(h) \geq 0$.
5. Succession effect: $\delta k(\theta_m - \theta_n^*(h))\left(\frac{de_m}{dh} - \frac{de_n}{dh} + 1\right)g(e_m^* - e_n^* + h)$, which is nonpositive if and only if $\theta_n^*(h) \geq \theta_m$.

As observed in the one-period model, the head start partially insulates the hiring manager from competition and leads him to hire an otherwise higher-ability agent, who could be promoted for the next period with positive probability. This *extended hiring effect* has a nonnegative impact on the principal's continuation payoff. Conversely, the head start also increases the probability of promoting the manager for the future period. Such a *succession effect* can be detrimental to future profit if and only if the manager's ability is lower than the new hire's.

Alternative Interpretation. It is worth noting that when $\theta_n^*(h) \geq \theta_m$, the term $\delta(\tilde{V}(\theta_m) - \tilde{V}(\theta_n^*(h)))G(e_m - e_n + h) < 0$ can also be viewed as the cost of providing a head start, which arises when the weaker player wins the tournament, as it potentially damages the firm's reputation and lowers employee morale. Therefore, the succession effect can be interpreted as the reputation or morale effect.

Proposition C.1. *In the two-period model, the optimal head start may allow for hiring sabotage in equilibrium (i.e., $\theta_n^*(h^*) < \bar{\theta}$) when the future stake is large ($\delta k > 0$). However, the new hire is always strictly better than the manager when sabotage arises in equilibrium.*

Proof. To show the existence of hiring sabotage in equilibrium with the optimal head start, we show $\theta_n^*(h^*) < \bar{\theta}$. When $\delta = 0.85$, $k = 16.4$, $M = 2.55$, $\tilde{v} = 1$, $\theta_m \in (0, 0.6)$, and $\bar{\theta} = 1$, we have $\theta_n^* < \bar{\theta}$.

Now we prove that $\theta_n^*(h^*) < \bar{\theta}$ implies $\theta_n^*(h^*) > \theta_m$ by contradiction. Suppose that $\theta_n^*(h^*) < \bar{\theta}$ and $\theta_n^*(h^*) \leq \theta_m$. Then, the succession effect is nonnegative, and the first-period effects are positive. Then, the succession effect is nonnegative, and the first-period effects are positive. Thus, equation (C.11) implies $d\tilde{\Pi}/dh > 0$, so increasing θ_n^* by increasing h would strictly increase $\tilde{\Pi}$, which contradicts the optimality of h^* . \square

In sharp contrast to the benchmark one-period model, in the two-period model where the winner's ability matters, the optimal head start may allow for hiring sabotage in equilibrium. This is driven by the *succession effect* of the head start. At the level of the head start that eliminates hiring sabotage (i.e., $h = \bar{h}$), if the (negative) succession effect dominates the sum of the aggregate positive effect on the first-period profit and the extended hiring effect on the continuation profit, the principal will lower the head start to increase the profit, which opens the room for hiring sabotage. This only happens when the manager's ability is lower than the new hire's because the succession effect would be positive otherwise. Therefore, regardless of the existence of hiring sabotage, the optimal contract ensures that the new hire is always of higher ability than the manager (i.e., $\theta_n^* \in (\theta_m, \bar{\theta}]$).²³

²³If the optimal contract allows for hiring sabotage, the new hire must be of higher ability than the manager. If it does not, then the new hire is $\theta_n = \bar{\theta}$. In either case, $\theta_n^* \in (\theta_m, \bar{\theta}]$.

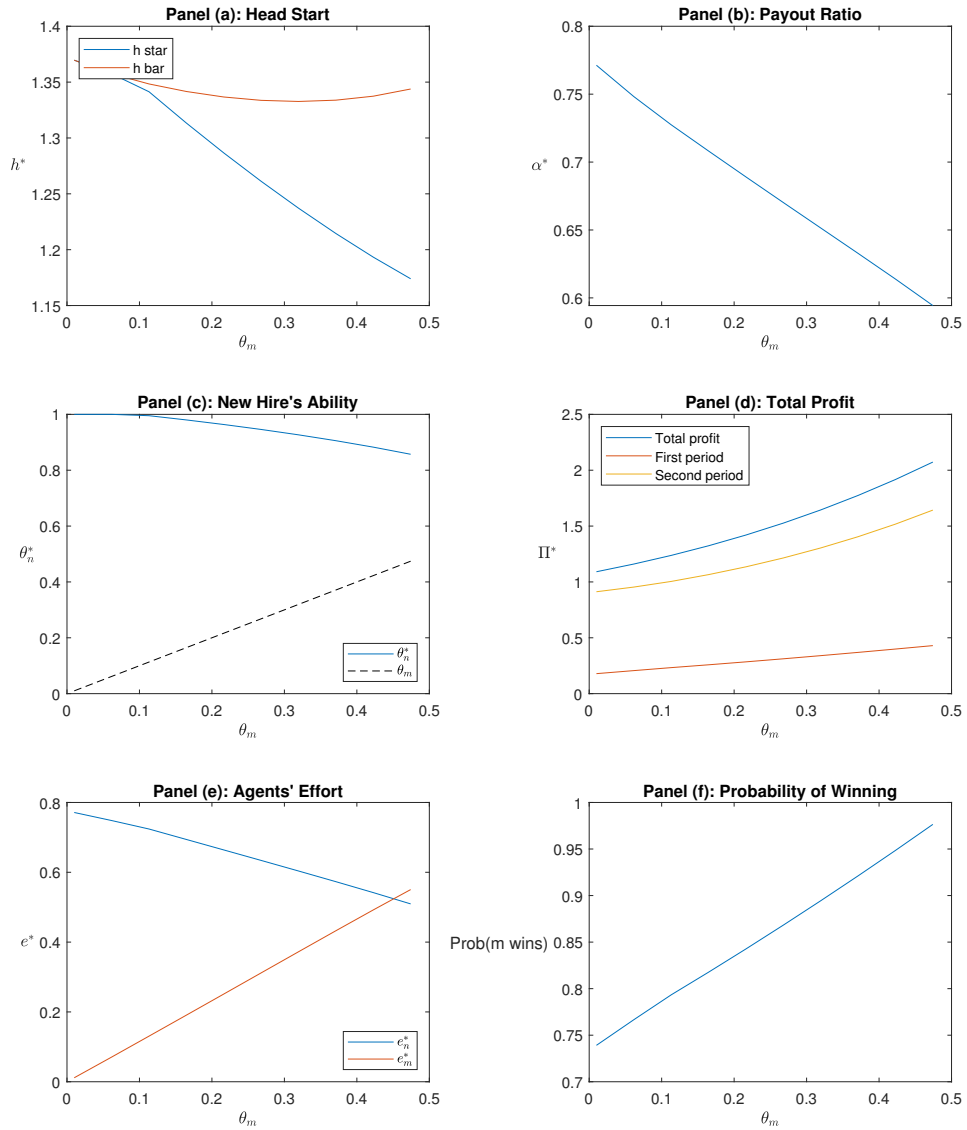


Figure C.1: Hiring sabotage arises due to continuation payoffs

Figure C.1 illustrates the optimal head start h^* , optimal payout ratio α^* , and the new hire's ability θ_n^* as a function of the manager's ability $\theta_m \in (0, 0.5)$ when $\delta = 0.85$, $\bar{\theta} = 1$, $k = 4$, $M = 2.55$, and $\tilde{v} = 1$.²⁴ It can be seen in panel (a) that the optimal head start h^* (blue line) is smaller than the sabotage-free level \bar{h} (red line), which allows for hiring sabotage—the new hire's ability θ_n^* (blue line), as shown in panel (c), is lower than $\bar{\theta} = 1$. However, it is still higher than the manager's ability (45° dashed line).

Moreover, it can be deduced from panels (a) and (b) that an increase in the manager's ability θ_m *decreases* both the optimal head start h^* and the optimal payout ratio α^* (blue line), in contrast to the one-period model. This is because the agents now have career incentives (continuation payoffs v) in addition to the first-period tournament prize incentives. As the manager's ability increases, the career incentive encourages him to invest more effort, thereby allowing the principal to lower the optimal payout ratio α^* , which in turn, lowers the optimal head start h^* . As the manager's ability θ_m increases, the decrease in the optimal head start h^* and the optimal payout ratio α^* have opposite effects on hiring sabotage—the former exacerbates it and the latter mitigates it. In combination, the former dominates the latter, and they jointly lead to a decrease in the new hire's ability θ_n^* , as shown in panel (c). Nevertheless, the firm's total profit is still increasing in the manager's ability θ_m , as shown in panel (d), because the direct effect of a better manager on the profit outweighs the indirect effect due to the increase in hiring sabotage.

Declaration of generative AI and AI-assisted technologies in the writing process

Statement: During the preparation of this work, the authors used ChatGPT for copy editing and proof checking. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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²⁴For $\theta_m > 0.5$, the probability of winning is 1, as shown in panel (f), so a pure-strategy equilibrium does not exist.

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